# EXCISION TECHNIQUE IN CONSTRAINED FORMULATIONS OF EINSTEIN EQUATIONS

Journée Gravitation et Physique Fondamentale

Meudon, 27 May 2014

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# Outline

· Einstein equations.

· Black holes in numerical simulations:

- Excision technique in dynamical simulations: spherically symmetric case.

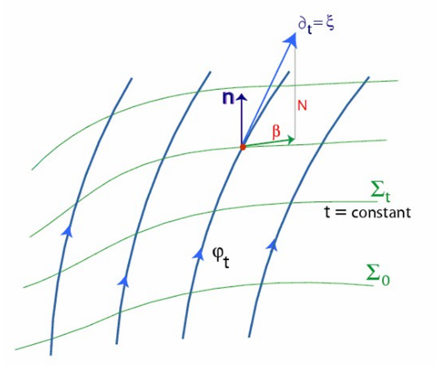
- Numerical results.

· Conclusions and future work.

# Einstein equations

We need a relativistic treatment of gravity in high density and strong curvature regimes, for example when black holes are considered.

3+1 formalism (Lichnerowitz 1944, Choquet-Bruhat 1952): spacetime is foliated by spacelike hypersurfaces, and an evolution through different hypersurfaces is performed.




Gauge freedom: lapse function and shift vector (4 variables) can be freely chosen to consider the specific foliation of spacetime.

Einstein equations: evolution equations (6 + 6) (hyperbolic) and constraint equations (4) (elliptic).

CFC (Conformally Flat Condition): Isenberg, 1979/2008; Wilson and Mathews, 1989.

- Approximation to Einstein equations: the tensor containing the gravitational radiation is neglected.
- Exact in spherical symmetry (C.-C. et al., 2011, constructive proof). Very accurate for axisymmetric rotating neutron stars.
- Set of elliptic equations (including constraint equations).

# Black holes in numerical simulations

· Dynamical approach: Collapse scenario:

$\mathbb{R}^3 \times \mathbb{R}^+ \longrightarrow$ formation of black hole and apparent horizon $\longrightarrow (\mathbb{R}^3 - \mathcal{B}) \times \mathbb{R}^+$

· Excision technique: remove a topological sphere containing the singularity and impose boundary conditions at the excised surface.

· Hyperbolic equations: local velocities and propagation directions can be used to derive the boundary conditions.

· Elliptic equations: wrong boundary conditions invalidate the solution in the whole numerical domain.

· Freedom for the gauge variables at the boundary conditions. Derived conditions for all the rest variables.

· New approach for CFC in spherical symmetric spacetimes (C.-C., Novak, Vasset, Jaramillo, 2013, preprint).

Boundary conditions for the elliptic equations:

$$b = \text{constant} \qquad h^{ij} = 0 \longrightarrow \text{lapse function}$$

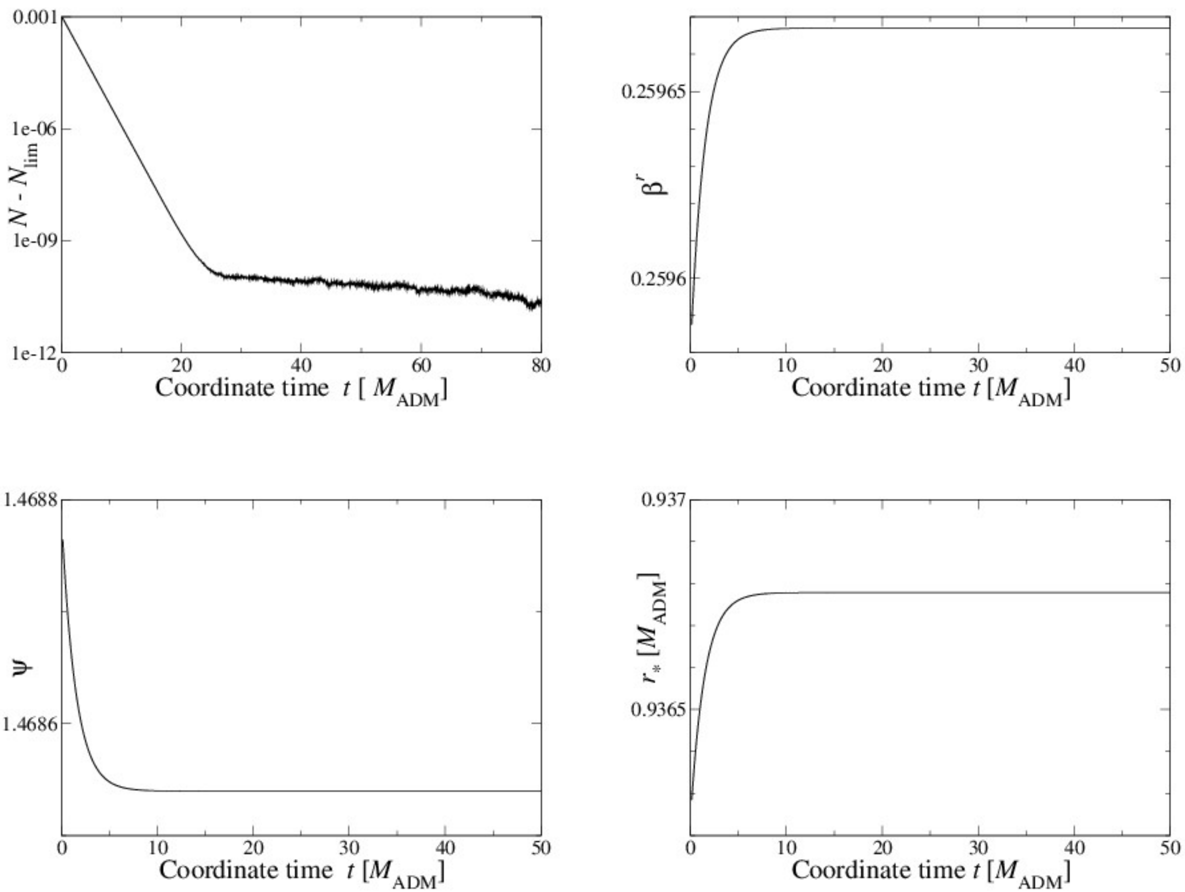
$$\partial_t \psi = \beta^k \mathcal{D}_k \psi + \frac{\psi}{6} \mathcal{D}_k \beta^k$$

# Numerical results

· Schwarzschild BH:

Initial boundary values: $\theta_0^{(l)} = -0.01, \quad N = 0.55, \quad b - N = 0.01$

$M_{\rm ADM} \simeq 1.09 M_\odot$ and AH located at $r \simeq 0.94 M_{\rm ADM}$




$|\delta b_1| \leq 4 \cdot 10^{-3}$

$p \simeq 1.01$

Stable evolution
$(t \sim 1000)$

Mass conservation similar to conservation of the AH surface (error $\sim 10^{-9}$)

Second-order convergence

# Numerical results

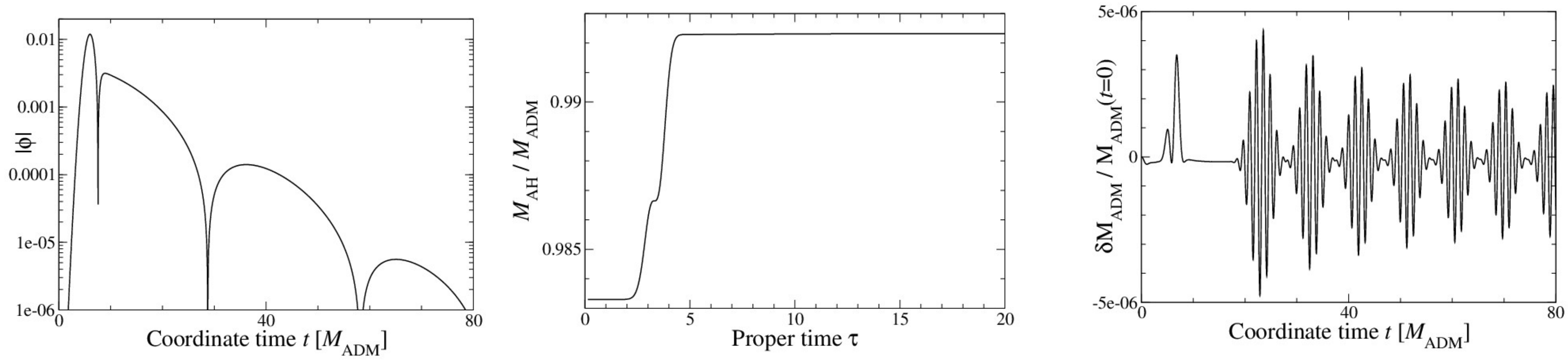
· Accretion of a massless scalar field:

$$\nabla^\mu \nabla_\mu \phi = 0 \longrightarrow T_{\mu\nu} = \nabla_\mu \phi \nabla_\nu \phi - \frac{1}{2}\gamma_{\mu\nu} \nabla_\rho \phi \nabla^\rho \phi$$

The wave equation is rewritten as a first order system. Sommerfeld-like condition is imposed at the outer boundary. No boundary conditions needed at the excision surface (all characteristic directed out of the computational domain as long as $b - N > 0$).

Initial data given by a Gaussian profile (for r>1):

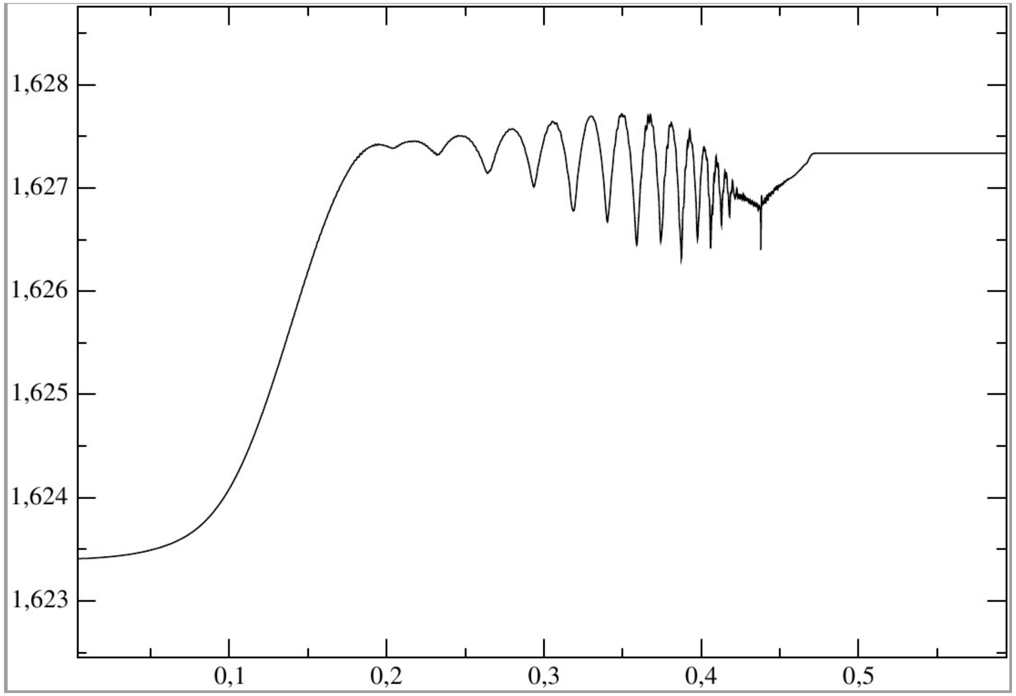
$$\phi(r, t=0) = \frac{\phi_0\, r^2}{1+r^2}\left(e^{-(r-r_0)^2/\sigma^2} + e^{-(r+r_0)^2/\sigma^2}\right), \quad \phi_0 = 0.01, \quad r_0 = 5, \quad \sigma = 1$$



# Numerical results

· Collapse of a neutron star to a black hole using the CoCoNuT code: dynamical evolution of matter content and spacetime, complex microphysics, magnetic fields...

Similar results as in the simplified models: stable evolution, convergence to coordinates adapted to stationary, quite accurate ADM mass conservation.



# Conclusions and future work

· General Relativity: appropriate tool to include in numerical simulations of astrophysical scenarios containing compact objects (e.g., black holes).

· Excision technique in constrained formulations to avoid the description of the physical singularity in the numerical domain: new approach in progress.

- First step: spherical symmetric spacetimes. Tested numerically. We have theoretical arguments in agreement with the numerical results. Implementation in the CoCoNuT code (Jerome Novak).

- Second step: Generalization to general spacetimes without symmetries (3D case). Work in progress: some ideas not yet tested numerically. Implementation should be quite direct from previous step.

Thank you for your attention!!