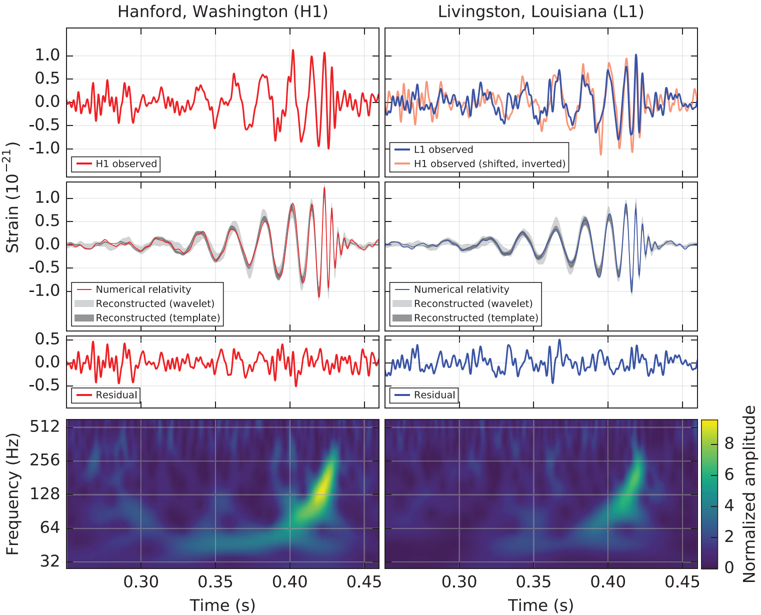
Orbital Dynamics of Eccentric Compact Binaries

Alexandre Le Tiec

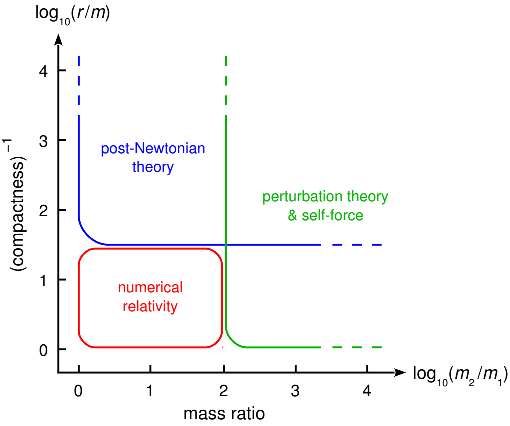
Laboratoire Univers et Théories
Observatoire de Paris / CNRS

Collaborators: S. Akcay, L. Barack, N. Sago, N. Warburton

Phys. Rev. D **91** 124014 (2015), arXiv:1503.01374 [gr-qc]
Phys. Rev. D **92** 084021 (2015), arXiv:1506.05648 [gr-qc]

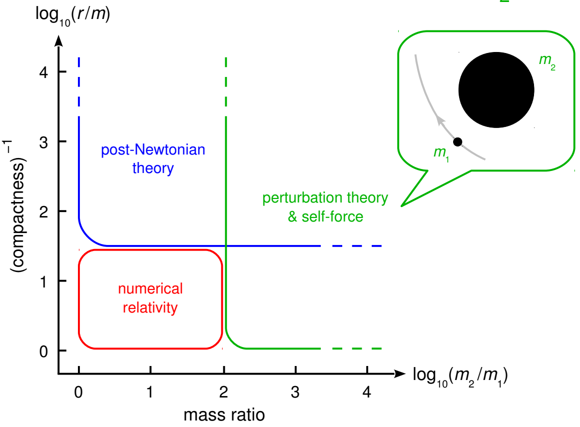

Hanford, Washington (H1)
Livingston, Louisiana (L1)
H1 observed
L1 observed
H1 observed (shifted, inverted)
Strain (10^{-21})
Numerical relativity
Reconstructed (wavelet)
Reconstructed (template)
Residual
Frequency (Hz)
Time (s)
Normalized amplitude

Source modelling for compact binaries

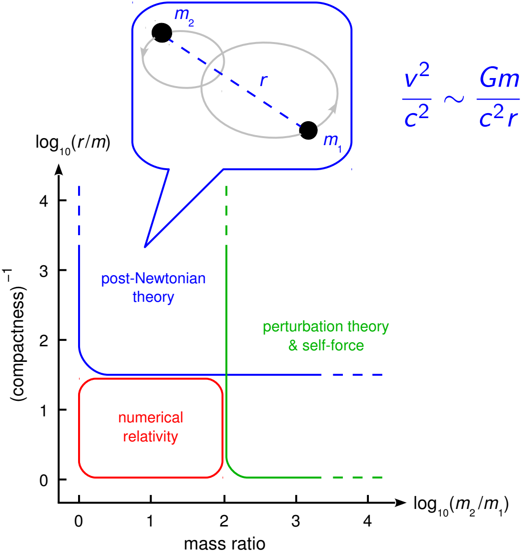


Source modelling for compact binaries

$$q \equiv \frac{m_1}{m_2} \ll 1$$

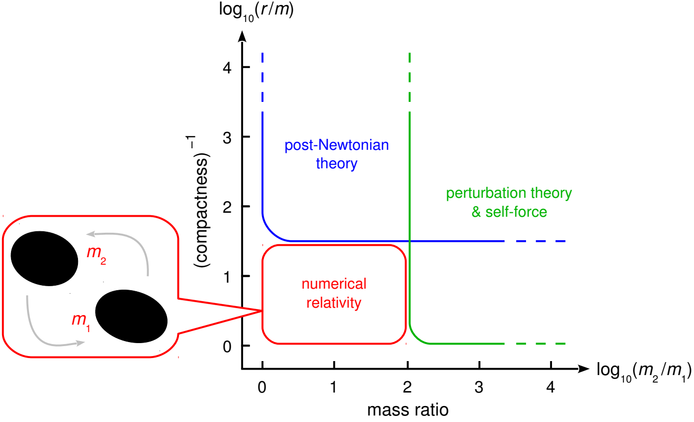


Source modelling for compact binaries

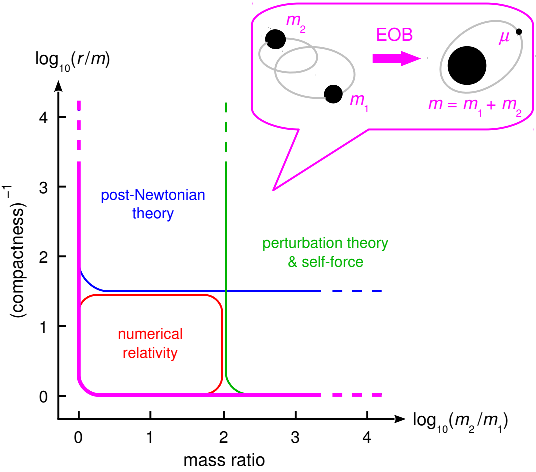


$$\frac{v^2}{c^2} \sim \frac{Gm}{c^2 r} \ll 1$$

Source modelling for compact binaries



Source modelling for compact binaries



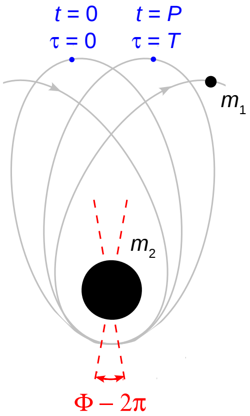
Averaged redshift for eccentric orbits

- Generic eccentric orbit parameterized by the two frequencies

$$n = \frac{2\pi}{P}\,, \quad \omega = \frac{\Phi}{P}$$

- Time average of redshift $z = \mathrm{d}\tau/\mathrm{d}t$ over one radial period

$$\langle z \rangle \equiv \frac{1}{P}\int_0^P z(t)\,\mathrm{d}t = \frac{T}{P}$$



First law of binary mechanics

- Canonical ADM Hamiltonian H of two point masses m_a
- Variation δH + Hamilton's equation + orbital averaging:

$$\delta M = \omega \, \delta L + n \, \delta R + \sum_a \langle z_a \rangle \, \delta m_a$$

- First integral associated with the variational first law:

$$M = 2 \, (\omega L + n R) + \sum_a \langle z_a \rangle m_a$$

- These relations are satisfied up to *at least* 3PN order

Applications of the first law

- Conservative dynamics beyond the geodesic approximation
- Shift of the Schwarzschild separatrix and singular curve
- Calibration of EOB potentials for generic bound orbits

$$\frac{\partial M}{\partial m_1} = \langle z \rangle - \omega \frac{\partial \langle z \rangle}{\partial \omega} - n \frac{\partial \langle z \rangle}{\partial n}$$
$$\frac{\partial L}{\partial m_1} = -\frac{\partial \langle z \rangle}{\partial \omega}$$
$$\frac{\partial R}{\partial m_1} = -\frac{\partial \langle z \rangle}{\partial n}$$

Applications of the first law

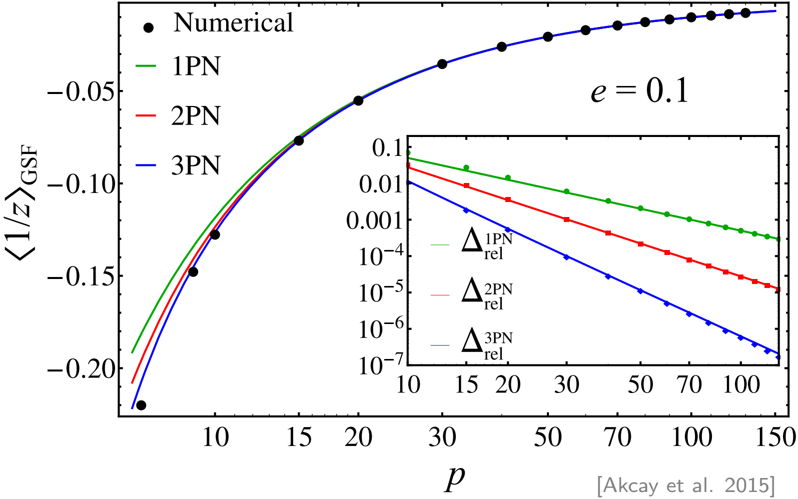
- Conservative dynamics beyond the geodesic approximation
- Shift of the Schwarzschild separatrix and singular curve
- **Calibration of EOB** potentials for generic bound orbits

$$\frac{\partial M}{\partial m_1} = \langle z \rangle - \omega \frac{\partial \langle z \rangle}{\partial \omega} - n \frac{\partial \langle z \rangle}{\partial n}$$

$$\frac{\partial L}{\partial m_1} = -\frac{\partial \langle z \rangle}{\partial \omega}$$

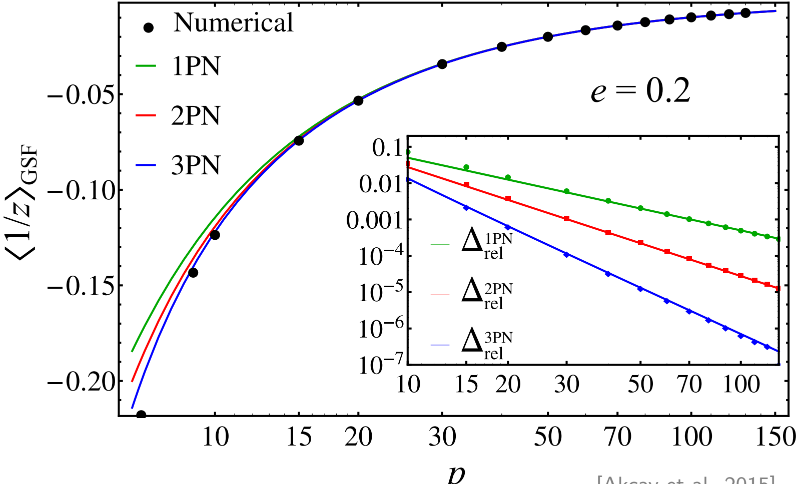
$$\frac{\partial R}{\partial m_1} = -\frac{\partial \langle z \rangle}{\partial n}$$

Averaged redshift *vs* semi-latus rectum



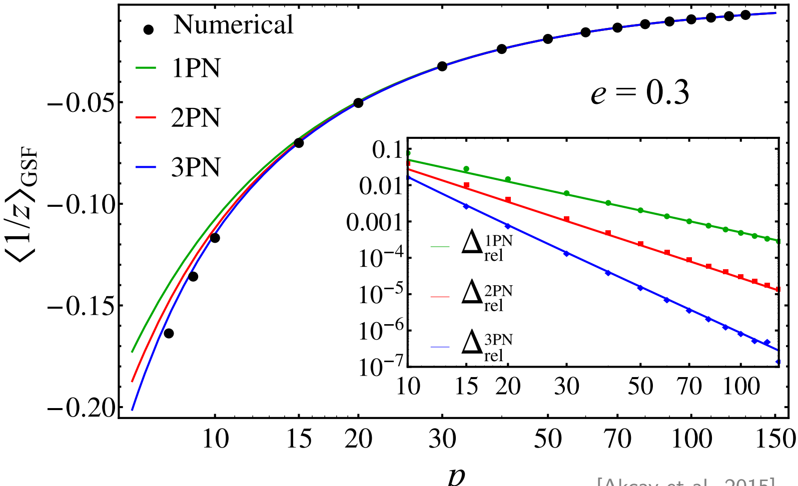
[Akcay et al. 2015]

Averaged redshift *vs* semi-latus rectum



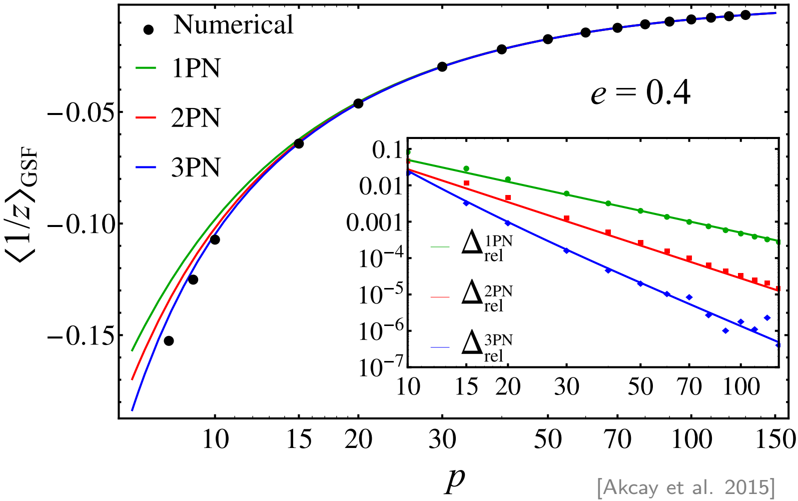
[Akcay et al. 2015]

Averaged redshift *vs* semi-latus rectum



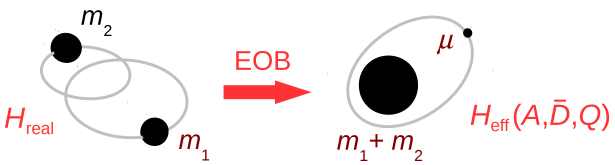
[Akcay et al. 2015]

Averaged redshift *vs* semi-latus rectum



[Akcay et al. 2015]

EOB dynamics beyond circular motion

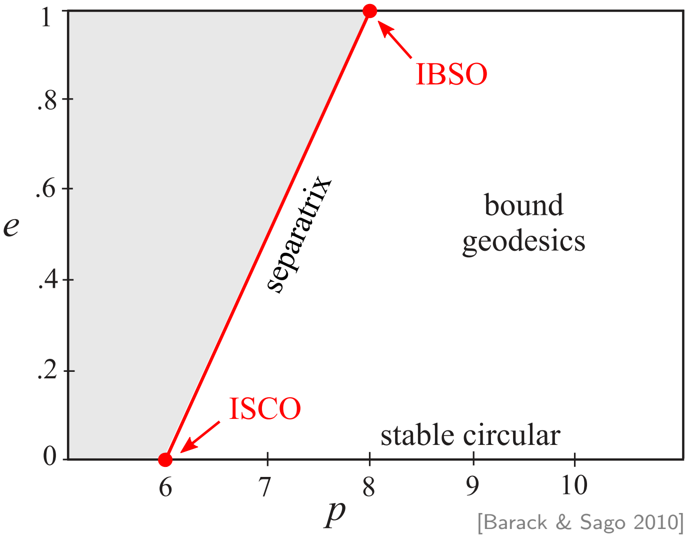


- Conservative EOB dynamics determined by "potentials"

$$A = 1 - 2M/r + \nu\, a(r) + \mathcal{O}(\nu^2)$$
$$\bar{D} = 1 + \nu\, \bar{d}(r) + \mathcal{O}(\nu^2)$$
$$Q = \nu\, q(r)\, p_r^4 + \mathcal{O}(\nu^2)$$

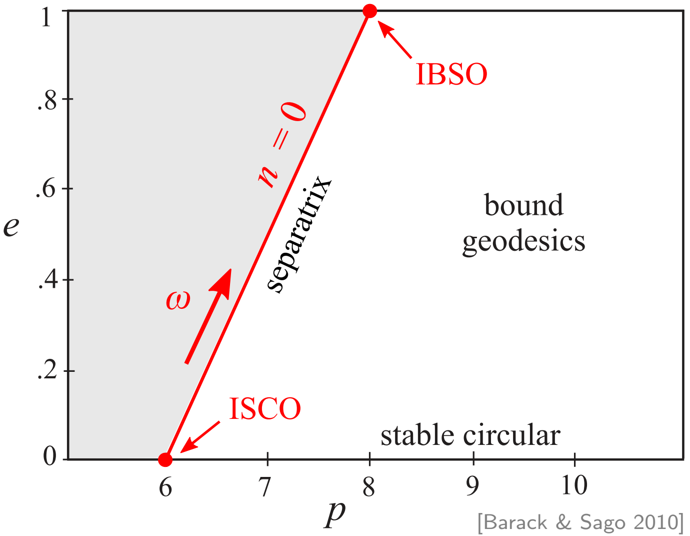
- Functions $a(r)$, $\bar{d}(r)$ and $q(r)$ controlled by $\langle z \rangle_{\mathrm{GSF}}(n, \omega)$

Schwarzschild separatrix



[Barack & Sago 2010]

Schwarzschild separatrix



[Barack & Sago 2010]

Shift of the Schwarzschild separatrix

- Separatrix $\omega = \omega_{\text{sep}}(e)$ characterized by the condition

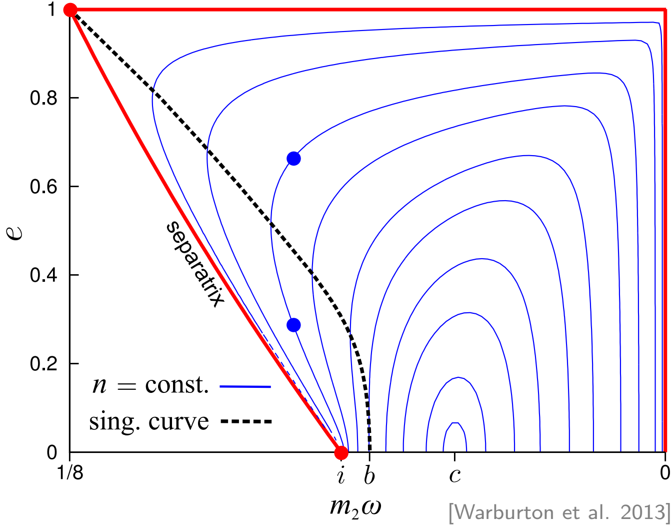
$$n = 0$$

- GSF-induced shift of Schwarzschild ISCO frequency
  [Barack & Sago 2009; Le Tiec et al. 2012; Akcay et al. 2012]

$$\frac{\Delta\omega_{\text{isco}}}{\omega_{\text{isco}}} = 1.2101539(4)\, q$$

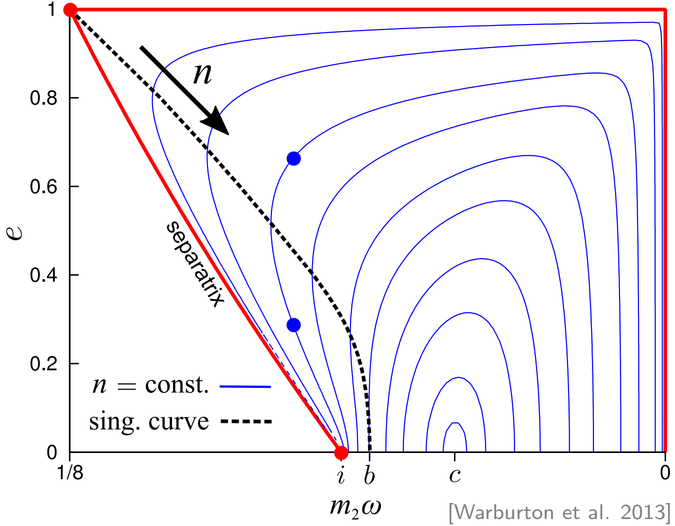
- GSF-induced shift of Schwarzschild IBSO frequency ?
- $\mathcal{O}(q)$ shift in $\omega = \omega_{\text{sep}}(e)$ controlled by $\langle z \rangle_{\text{GSF}}(n, \omega)$

Schwarzschild singular curve



[Warburton et al. 2013]

Schwarzschild singular curve



[Warburton et al. 2013]

Shift of the Schwarzschild singular curve

- Singular curve $\omega = \omega_{\text{sing}}(n)$ characterized by condition

$$\left| \frac{\partial(n,\omega)}{\partial(M,L)} \right| = 0$$

- In the test-particle limit $q \to 0$ this is equivalent to

$$\left[\left(\partial^2_{n\omega} \langle z \rangle \right)^2 - \partial^2_n \langle z \rangle \, \partial^2_\omega \langle z \rangle \right]^{-1} = 0$$

- $\mathcal{O}(q)$ shift in $\omega = \omega_{\text{sing}}(n)$ controlled by $\langle z \rangle_{\text{GSF}}(n,\omega)$

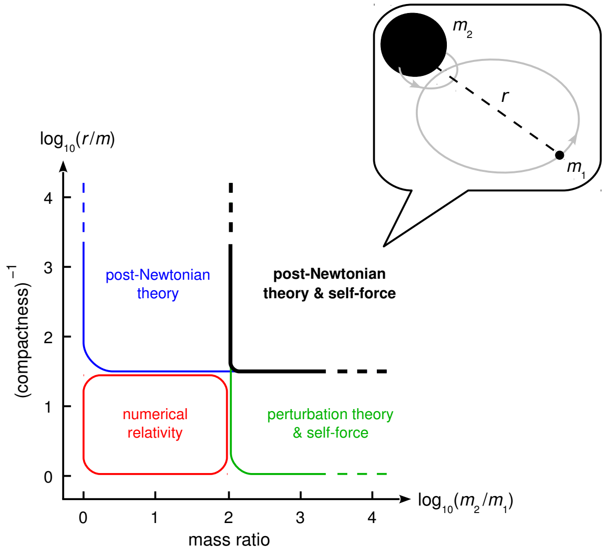
Summary

- First law of mechanics for eccentric-orbit compact binaries
- Self-force calculation of $\langle z \rangle(n, \omega)$ for eccentric orbits
- Numerous applications of the first law:
  - Conservative dynamics beyond the geodesic approximation
  - Shift of the Schwarzschild separatrix and singular curve
  - Calibration of EOB potentials for generic bound orbits
  - ...

Prospects

- Extension of the first law to precessing spinning binaries
- Self-force calculation of $\langle \psi \rangle(n, \omega)$ for eccentric orbits

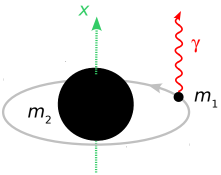
Additional Material


m_2
r
m_1
$\log_{10}(r/m)$
$(\text{compactness})^{-1}$
4
3
2
1
0
post-Newtonian theory
post-Newtonian theory & self-force
numerical relativity
perturbation theory & self-force
0
1
2
3
4
$\log_{10}(m_2/m_1)$
mass ratio

Redshift invariant for circular orbits

- It measures the redshift of light emitted from the point particle [Detweiler 2008]

$$\frac{\mathcal{E}_{\text{obs}}}{\mathcal{E}_{\text{em}}} = \frac{(p^a u_a)_{\text{obs}}}{(p^a u_a)_{\text{em}}} = z$$

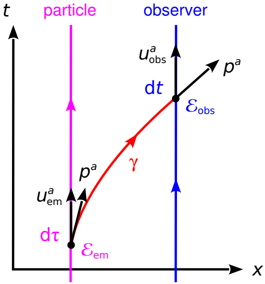


- It is a constant of the motion associated with the helical Killing field k^a:

$$z = -k^a u_a$$

- In coordinates adapted to the symmetry:

$$z = \frac{\mathrm{d}\tau}{\mathrm{d}t} = \frac{1}{u^t}$$



Extracting post-Newtonian coefficients

	Coeff.	Exact value [Akcay et al. 2015]	Fitted value [Akcay et al. 2015]	Fitted value [Meent, Shah 2015]
1PN	e^2	4	4.0002(8)	$4 \pm 6 \times 10^{-12}$
	e^4	-2	$-2.00(1)$	$-2 \pm 4 \times 10^{-10}$
	e^6	0		$0 \pm 4 \times 10^{-9}$

Extracting post-Newtonian coefficients

	Coeff.	Exact value [Akcay et al. 2015]	Fitted value [Akcay et al. 2015]	Fitted value [Meent, Shah 2015]
1PN	e^2	4	$4.0002(8)$	$4 \pm 6 \times 10^{-12}$
	e^4	-2	$-2.00(1)$	$-2 \pm 4 \times 10^{-10}$
	e^6	0		$0 \pm 4 \times 10^{-9}$
2PN	e^2	7	$7.02(2)$	$7 \pm 6 \times 10^{-9}$
	e^4	$1/4$		$1/4 \pm 4 \times 10^{-7}$
	e^6	$5/2$		$5/2 \pm 4 \times 10^{-6}$

Extracting post-Newtonian coefficients

	Coeff.	Exact value [Akcay et al. 2015]	Fitted value [Akcay et al. 2015]	Fitted value [Meent, Shah 2015]
1PN	e^2	4	4.0002(8)	$4 \pm 6 \times 10^{-12}$
	e^4	-2	$-2.00(1)$	$-2 \pm 4 \times 10^{-10}$
	e^6	0		$0 \pm 4 \times 10^{-9}$
2PN	e^2	7	7.02(2)	$7 \pm 6 \times 10^{-9}$
	e^4	$1/4$		$1/4 \pm 4 \times 10^{-7}$
	e^6	$5/2$		$5/2 \pm 4 \times 10^{-6}$
3PN	e^2	$-14.312097\cdots$	$-14.5(4)$	$-14.3120980(5)$
	e^4	$83.382963\cdots$		83.38298(7)
	e^6	$-36.421975\cdots$		$-36.421(3)$

Extracting post-Newtonian coefficients

	Coeff.	Exact value [Akcay et al. 2015]	Fitted value [Akcay et al. 2015]	Fitted value [Meent, Shah 2015]
1PN	e^2	4	$4.0002(8)$	$4 \pm 6 \times 10^{-12}$
	e^4	-2	$-2.00(1)$	$-2 \pm 4 \times 10^{-10}$
	e^6	0		$0 \pm 4 \times 10^{-9}$
2PN	e^2	7	$7.02(2)$	$7 \pm 6 \times 10^{-9}$
	e^4	$1/4$		$1/4 \pm 4 \times 10^{-7}$
	e^6	$5/2$		$5/2 \pm 4 \times 10^{-6}$
3PN	e^2	$-14.312097\cdots$	$-14.5(4)$	$-14.3120980(5)$
	e^4	$83.382963\cdots$		$83.38298(7)$
	e^6	$-36.421975\cdots$		$-36.421(3)$

New coefficients at 4PN and 5PN orders [van de Meent, Shah 2015]