Geons in Asymptotically Anti-de Sitter spacetimes

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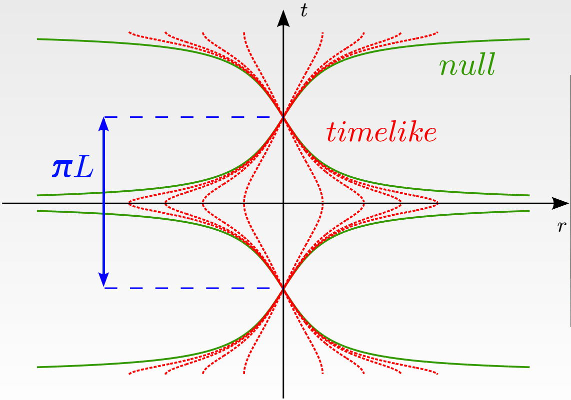
Anti-de Sitter (AdS) spacetime

Geodesics

AdS = unique maximally symmetric solution of Einstein with $\Lambda < 0$

$$R_{\mu\nu} = \Lambda g_{\mu\nu} \qquad \Lambda = -\frac{3}{L^2}$$



AdS/CFT correspondance

Holographic principle :
Strongly coupled 4D gauge theory = Grav. theory in 5D AAdS

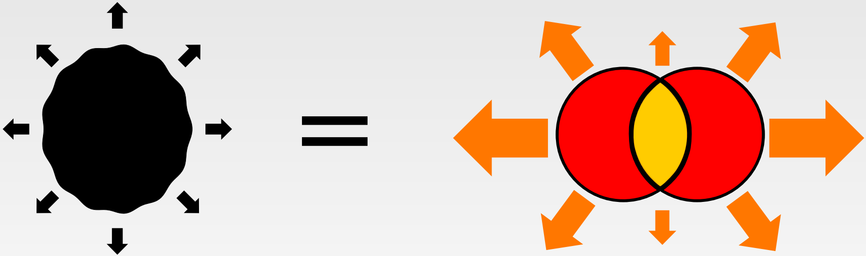
AdS/CFT recipe

- Solve Einstein equation in *AdS*
- Euclidean action $\underline{S_E}$ on the AdS boundary
- GKP-Witten : $Z_{\mathrm{CFT}} \simeq exp(-\underline{S_E})$

$AdS_5 \Leftrightarrow \mathcal{N} = 4$ SYM $\Leftrightarrow$ proxy for QCD

AdS/CFT correspondance

- One example (out of many) : Damping time ⇔ viscosity



Schwarzschild   Quark-gluon plasma

Stability of AdS

Conjecture : Any perturbation of AdS is doomed to collapse to a black hole

Observation
(Bizon and Rostworowski 2011) :

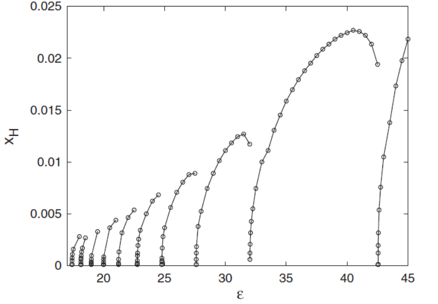


FIG. 1. Horizon radius vs amplitude for initial data (9). The number of reflections off the AdS boundary before collapse varies from zero to nine (from right to left).

Formation of Black Hole $\boxed{t = O(\varepsilon^{-2})}$.

Explaination :

$$R_{\alpha\beta} = \Lambda g_{\alpha\beta}$$

Perturbative expansion :

$$y = y_0 + \varepsilon y_1 + \varepsilon^2 y_2 + \dots$$

Third order equation :

$$y_3'' + \omega_j^2 y_3 = A\cos(\omega_j t) + \dots$$

$$\Rightarrow y_3 = \frac{A}{2\omega_j} t \sin(\omega_j t) + \dots$$

Secular resonance !

Breakdown at $\boxed{t = O(\varepsilon^{-2})}$.

Question : Are there solutions which *a priori* never collapse to black holes ?

yes : **GEON = Electro-Gravitational Entity**
(originally coined by J.A. Wheeler in 1955)

Different kinds of geons

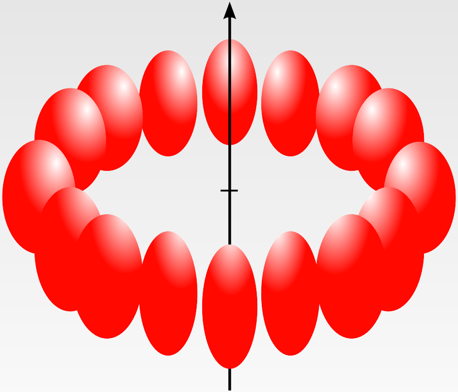
- **Massive boson :** boson stars (Buchel et al 2013)
- **Massless boson :** scalar breathers (Fodor et al 2015)
- **Massive vector :** Proca stars (Brito et al 2015)
- **Massless vector :** Maxwell solitons (Heirdeiro and Radu 2016)
- ***Gravitational geons :*** (Horowitz and Santos 2015)

Gravitational geons in AAdS spacetime

What is a gravitational geon ?

GW packet

- Self-gravitating
- Self-rotating
- Self-interacting



How to build a geon ?

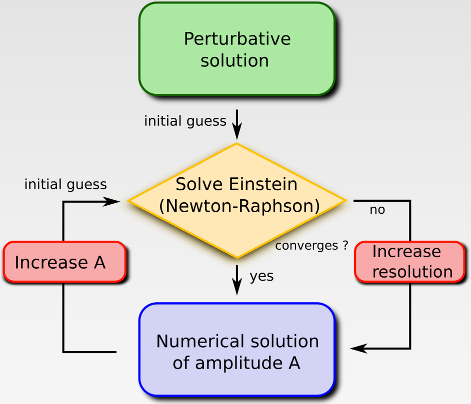
(Dias et al 2012)

- Look for periodic GW
- Choose the Y^l_m you prefer
- Perturbative expansion

$$y = y_0 + \varepsilon y_1 + \varepsilon^2 y_2 + \dots$$
$$y_3'' + \omega^2 y_3 = A\cos(\omega t) + \dots$$

$\Rightarrow$ BAD !

- Poincaré-Linstedt method :
  Promote $\omega = \omega_0 + \varepsilon^2 \omega_2 + \dots$
  $\Rightarrow$ GOOD ! (sometimes)
- Look for helical symmetry
  Killing vector $\partial_\tau = \partial_t + \omega \partial_\varphi$
  $\Rightarrow$ AWESOME !



- 11 unknowns : $g_{\alpha\beta}$ and ω
- 11 equations : $G_{\alpha\beta}$ and fixed A

What about gauge freedom ?

- We look for the metric $g_{\alpha\beta}$: **10 unknowns**
- Einstein equation $R_{\alpha\beta} = \Lambda g_{\alpha\beta}$:
  Naively 10 equations but **only 6 are independent**

$$\boxed{10 - 6 = 4}$$

$\Rightarrow$ 4 choices of coordinates (x^0, x^1, x^2, x^3)

What do we want ?

A system of 10 independant equations equivalent to Einstein that fixes the coordinates

Consider 3+1 formalism, the 3D metric γ_{ij} and the 3D Ricci tensor :

$$R_{ij} = -\frac{\gamma^{kl}}{2}\left(\underbrace{\partial_k\partial_l\gamma_{ij}}_{\text{laplacian}} + \underbrace{\partial_i\partial_j\gamma_{kl} - \partial_i\,\partial_l\gamma_{jk} - \partial_j\partial_k\gamma_{il}}_{\text{BAD !}}\right) + \underbrace{\gamma^{kl}\gamma_{mn}(\Gamma^l_{jk}\Gamma^k_{il} - \Gamma^m_{kl}\Gamma^n_{ij})}_{1^{st}\text{ order derivatives}}$$

But if we introduce $\boxed{V^i = \gamma^{kl}(\Gamma^i_{kl} - \bar{\Gamma}^i_{kl})}$ and $\boxed{V_{ij} = D_{(i}V_{j)}}$, then :

$$R_{ij} - V_{ij} = \text{laplacian} + 1^{st}\text{ order derivatives}$$

We want $K = 0$ and $V^i = 0$ (**4 gauge conditions**), so we solve :

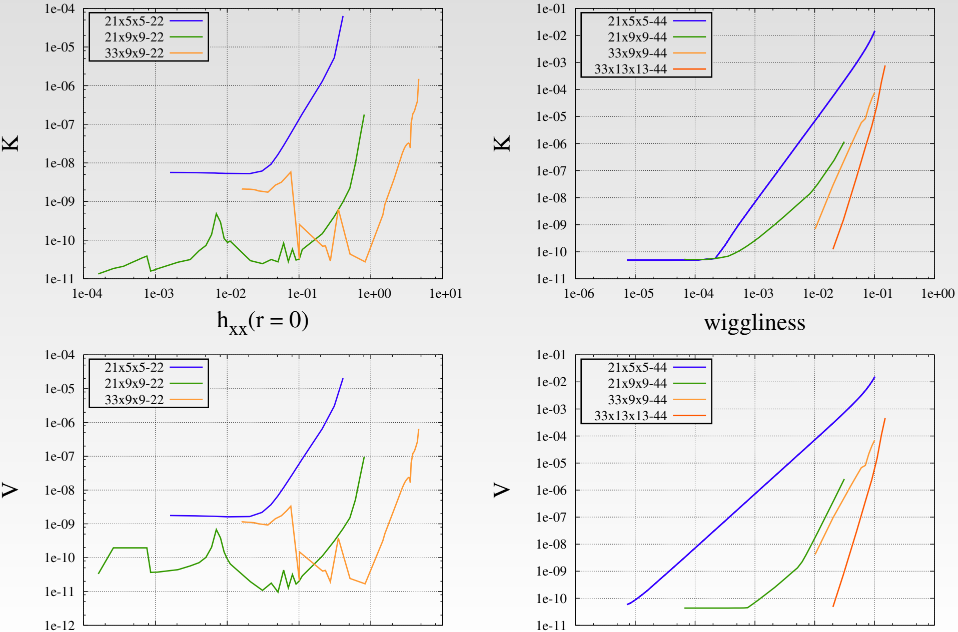
$$R - D_iV^i + \cancel{K^2} - K_{ij}K^{ij} - 2\Lambda = 0$$

$$D_jK^j_i - \cancel{D_iK} = 0$$

$$\mathcal{L}_\beta K_{ij} - D_iD_jN + N\left[R_{ij} - V_{ij} + \cancel{KK_{ij}} - 2K_{ik}K^k_j - \Lambda\gamma_{ij}\right] = 0$$

and check *a posteriori* that K and V^i are indeed zero.
(Andersson and Moncrief 2003)

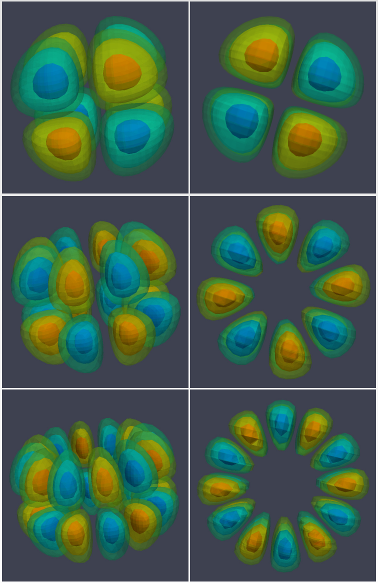
geon seed : Y_2^2 | geon seed : Y_4^4



Families of geon

$$\hat{h}_{\alpha\beta} = \Omega^2 (g_{\alpha\beta} - \bar{g}_{\alpha\beta})$$

Pictured : $\hat{h}_{0z}$



angular radial

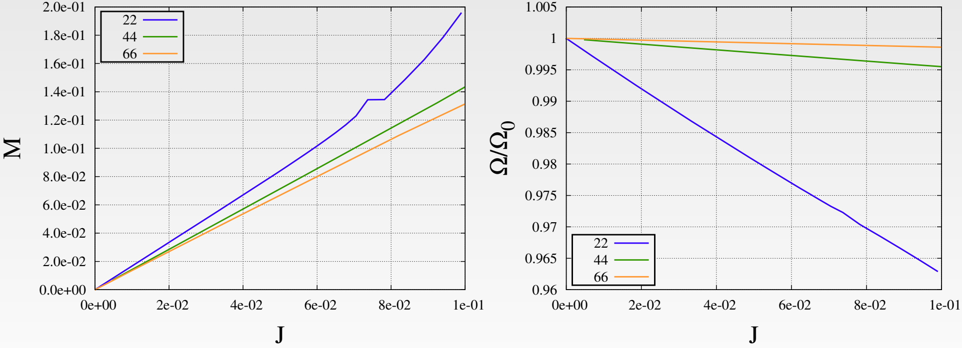
(2,2,0)

(4,4,0)

(6,6,0)

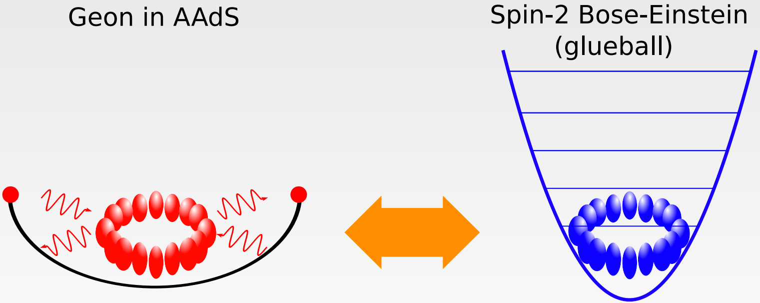
Global quantities

Typical lenght scale : $L = 1$



AdS/CFT interpretation

- Dual representation : No black hole ⇒ $\boxed{T = 0}$



Conclusion

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TAKE AWAY INFORMATION :

- AdS is interesting because of the AdS/CFT correspondance
- AdS is unstable and any perturbative approach breaks down at 3^{rd} order. . .
- . . .except in the case of geons
- We numerically constructed several families of gravitational geons

QUESTIONS :

- Are geons stable ?
- Is there a maximum mass ?
- Do excited states exist ?