Two-body problem in Scalar-Tensor theories, an Effective-One-Body approach

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June 19th 2017

arXiv:1703.05360 FLJ - Nathalie Deruelle

Motivations

• GW150914 : The very first observation of a BBH coalescence by LIGO-Virgo has opened **a new era in gravitational wave astronomy.**

• Opportunity to bring **new tests of modified gravities**, in the strong-field regime near merger, a topic which is for the moment still in infancy.

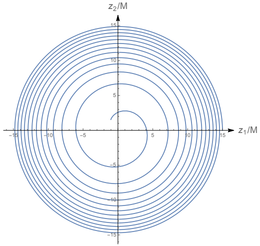
In General Relativity, "Effective-One-Body" (EOB) :

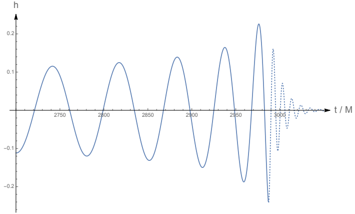
• Map the two-body PN dynamics to the motion of a **test particle** in an **effective SSS metric** [Buonanno-Damour 98]

$$H(Q,P)\ ,\quad \epsilon=\left(\frac{v}{c}\right)^2 \qquad \longrightarrow \qquad H_e(q,p)\ ,\quad ds_e^2=g^e_{\mu\nu}dx^\mu dx^\nu$$

$$\boxed{H_e=f_{\rm EOB}(H)}$$

• Defines a resummation of the PN dynamics, hence describes **analytically** the coalescence of 2 compact objects in **General Relativity**, from inspiral **to merger**.





• Instrumental to build libraries of waveform templates for LIGO-VIRGO

Motivations

Our proposition [arXiv:1703.05360]

- Can we extend the EOB approach to modified gravities ?
- Consider the simplest and most studied example of **massless Scalar-Tensor theories**.

• First buiding block : map the conservative part of the two-body dynamics onto the geodesic of an effective metric.

• ST-extension of [Buonanno-Damour 98]

We adopt the conventions of Damour and Esposito-Farèse [DEF 92, 95]

ST action in the Einstein-frame ($G_* \equiv c \equiv 1$)

$$S_{EF} = \frac{1}{16\pi} \int d^4x \sqrt{-g} \left(R - 2 g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi \right) + S_m \left[\Psi, \mathcal{A}^2(\varphi) g_{\mu\nu} \right]$$

When self-gravity is not negligible (neutron stars, black holes),

$$S_m = -\sum_A \int d\lambda \sqrt{-g_{\mu\nu} \frac{dx^\mu}{d\lambda} \frac{dx^\nu}{d\lambda}}\, m_A(\varphi)$$

$m_A(\varphi)$ depends on the theory $\mathcal{A}(\varphi)$ and on the EOS of body A.

$\rightarrow$ SEP violation [Eardley 75, DEF 92]

I) THE TWO – BODY HAMILTONIAN AT 2PK ORDER

Our starting point : what is known today

Two-body Scalar-Tensor Lagrangian [DEF 93][Mirshekari, Will 13]

- Harmonic coordinates $\partial_\mu(\sqrt{-g}g^{\mu\nu}) = 0$
- conservative 2PK dynamics : $\mathcal{O}((\frac{v}{c})^4) \sim \mathcal{O}((\frac{m}{r})^2)$ corrections to Kepler
- Weak field expansion

$$\begin{aligned} g_{\mu\nu} &= \eta_{\mu\nu} + \delta g_{\mu\nu} \\ \varphi &= \varphi_0 + \delta\varphi \end{aligned}$$

- the fundamental functions $m_A(\varphi)$ and $m_B(\varphi)$ are expanded around φ_0 :

$$\begin{aligned} \ln m_A(\varphi) &\equiv \ln m_A^0 + \alpha_A^0(\varphi-\varphi_0) + \beta_A^0(\varphi-\varphi_0)^2 + {\beta'}_A^0(\varphi-\varphi_0)^3 + \cdots \\ \ln m_B(\varphi) &\equiv \ln m_B^0 + \alpha_B^0(\varphi-\varphi_0) + \beta_B^0(\varphi-\varphi_0)^2 + {\beta'}_B^0(\varphi-\varphi_0)^3 + \cdots \end{aligned}$$

i.e. the 2PK Lagrangian depends on 8 fundamental **parameters**.

The two-body Lagrangian

Two-body 2PK Lagrangian

$$L = -m_A^0 - m_B^0 + L_{\mathrm{K}} + L_{1\mathrm{PK}} + L_{2\mathrm{PK}} + \cdots$$

$$\vec{N} \equiv \frac{\vec{Z}_A - \vec{Z}_B}{R}\,, \quad \vec{V}_A \equiv \frac{d\vec{Z}_A}{dt}\,, \quad R \equiv \mid \vec{Z}_A - \vec{Z}_B \mid\,, \quad \vec{A}_A \equiv \frac{d\vec{V}_A}{dt}$$

- Keplerian order :

$$L_{\mathrm{K}} = \frac{1}{2} m_A^0 V_A^2 + \frac{1}{2} m_B^0 V_B^2 + \frac{G_{AB} m_A^0 m_B^0}{R} \qquad \text{where} \qquad G_{AB} \equiv 1 + \alpha_A^0 \alpha_B^0$$

- post-Keplerian (1PK) :

$$\begin{aligned} L_{1\mathrm{PK}} &= \frac{1}{8} m_A^0 V_A^4 + \frac{1}{8} m_B^0 V_B^4 \\ &+ \frac{G_{AB} m_A^0 m_B^0}{R} \left(\frac{3}{2}(V_A^2 + V_B^2) - \frac{7}{2} \vec{V}_A \cdot \vec{V}_B - \frac{1}{2} (\vec{N} \cdot \vec{V}_A)(\vec{N} \cdot \vec{V}_B) + \bar{\gamma}_{AB} (\vec{V}_A - \vec{V}_B)^2 \right) \\ &- \frac{G_{AB}^2 m_A^0 m_B^0}{2R^2} \left(m_A^0 (1 + 2\bar{\beta}_B) + m_B^0 (1 + 2\bar{\beta}_A) \right) \end{aligned}$$

$$\text{where} \quad \bar{\gamma}_{AB} \equiv -\frac{2\alpha_A^0 \alpha_B^0}{1 + \alpha_A^0 \alpha_B^0} \qquad \bar{\beta}_A \equiv \frac{1}{2} \frac{\beta_A^0 (\alpha_B^0)^2}{(1 + \alpha_A^0 \alpha_B^0)^2} \quad (A \leftrightarrow B)$$

- post-post-Keplerian (2PK) :

$$
\begin{aligned}
L_{2\mathrm{PK}} &= \frac{1}{16} m_A^0 V_A^6 \\
&+ \frac{G_{AB} m_A^0 m_B^0}{R}\left[\frac{1}{8}(7+4\bar\gamma_{AB})\left(V_A^4 - V_A^2(\vec N\cdot\vec V_B)^2\right) - (2+\bar\gamma_{AB})V_A^2(\vec V_A\cdot\vec V_B) + \frac{1}{8}(\vec V_A\cdot\vec V_B)^2\right. \\
&\qquad \left. + \frac{1}{16}(15+8\bar\gamma_{AB})V_A^2V_B^2 + \frac{3}{16}(\vec N\cdot\vec V_A)^2(\vec N\cdot\vec V_B)^2 + \frac{1}{4}(3+2\bar\gamma_{AB})\vec V_A\cdot\vec V_B(\vec N\cdot\vec V_A)(\vec N\cdot\vec V_B)\right] \\
&+ \frac{G_{AB}^2 m_B^0 (m_A^0)^2}{R^2}\left[\frac{1}{8}\left(2+12\bar\gamma_{AB}+7\bar\gamma_{AB}^2+8\bar\beta_B-4\delta_A\right)V_A^2 + \frac{1}{8}\left(14+20\bar\gamma_{AB}+7\bar\gamma_{AB}^2+4\bar\beta_B-4\delta_A\right)V_B^2\right. \\
&\qquad - \frac{1}{4}\left(7+16\bar\gamma_{AB}+7\bar\gamma_{AB}^2+4\bar\beta_B-4\delta_A\right)\vec V_A\cdot\vec V_B - \frac{1}{4}\left(14+12\bar\gamma_{AB}+\bar\gamma_{AB}^2-8\bar\beta_B+4\delta_A\right)(\vec V_A\cdot\vec N)(\vec V_B\cdot\vec N) \\
&\qquad \left. + \frac{1}{8}\left(28+20\bar\gamma_{AB}+\bar\gamma_{AB}^2-8\bar\beta_B+4\delta_A\right)(\vec N\cdot\vec V_A)^2 + \frac{1}{8}\left(4+4\bar\gamma_{AB}+\bar\gamma_{AB}^2+4\delta_A\right)(\vec N\cdot\vec V_B)^2\right] \\
&+ \frac{G_{AB}^3 (m_A^0)^3 m_B^0}{2R^3}\left[1+\frac{2}{3}\bar\gamma_{AB}+\frac{1}{6}\bar\gamma_{AB}^2+2\bar\beta_B+\frac{2}{3}\delta_A+\frac{1}{3}\epsilon_B\right] + \frac{G_{AB}^3 (m_A^0)^2 (m_B^0)^2}{8R^3}\left[19+8\bar\gamma_{AB}+8(\bar\beta_A+\bar\beta_B)+4\zeta\right] \\
&- \frac{1}{8} G_{AB} m_A^0 m_B^0 \left(2(7+4\bar\gamma_{AB})\vec A_A\cdot\vec V_B(\vec N\cdot\vec V_B) + \vec N\cdot\vec A_A(\vec N\cdot\vec V_B)^2 - (7+4\bar\gamma_{AB})\vec N\cdot\vec A_A V_B^2\right) \\
&\qquad + (A \leftrightarrow B)
\end{aligned}
$$

where $\delta_A \equiv \frac{(\alpha_A^0)^2}{(1+\alpha_A^0\alpha_B^0)^2}$ $\quad \epsilon_A \equiv \frac{(\beta'_A\alpha_B^3)^0}{(1+\alpha_A^0\alpha_B^0)^3}$ $\quad \zeta \equiv \frac{\beta_A^0\alpha_A^0\alpha_B^0\beta_B^0}{(1+\alpha_A^0\alpha_B^0)^3}$ $(A \leftrightarrow B)$

The centre-of-mass two-body 2PK Hamiltonians

- Reduction by means of a **contact transformation**
- ordinary Hamiltonians :

$$\vec{P}_A = \frac{\partial L_f^{red}}{\partial \vec{V}_A} \; , \quad \vec{P}_B = \frac{\partial L_f^{red}}{\partial \vec{V}_B} \; , \quad H = \vec{P}_A \cdot \vec{V}_A + \vec{P}_B \cdot \vec{V}_B - L_f^{red}$$

- In the centre-of-mass frame : $\boxed{\vec{P}_A + \vec{P}_B \equiv \vec{0}}$

  i.e. $\vec{Z} \equiv \vec{Z}_A - \vec{Z}_B$ and $\vec{P} \equiv \vec{P}_A = -\vec{P}_B$

- The relative motion is planar $\rightarrow$ use polar coodinates $(Q, P) \equiv (R, \Phi, P_R, P_\Phi)$

The centre-of-mass two-body 2PK Hamiltonians

17 coefficients

$$H = M + \left(\frac{P^2}{2\mu} - \mu \frac{G_{AB} M}{R} \right) + H^{1\mathrm{PK}} + H^{2\mathrm{PK}} + \cdots$$

- $$\frac{H^{1\mathrm{PK}}}{\mu} = \left(h_1^{1\mathrm{PK}} \hat{P}^4 + h_2^{1\mathrm{PK}} \hat{P}^2 \hat{P}_R^2 + h_3^{1\mathrm{PK}} \hat{P}_R^4 \right) + \frac{1}{\hat{R}} \left(h_4^{1\mathrm{PK}} \hat{P}^2 + h_5^{1\mathrm{PK}} \hat{P}_R^2 \right) + \frac{h_6^{1\mathrm{PK}}}{\hat{R}^2}$$
- $$\frac{H^{2\mathrm{PK}}}{\mu} = \left(h_1^{2\mathrm{PK}} \hat{P}^6 + h_2^{2\mathrm{PK}} \hat{P}^4 \hat{P}_R^2 + h_3^{2\mathrm{PK}} \hat{P}^2 \hat{P}_R^4 + h_4^{2\mathrm{PK}} \hat{P}_R^6 \right)$$
$$+ \frac{1}{\hat{R}} \left(h_5^{2\mathrm{PK}} \hat{P}^4 + h_6^{2\mathrm{PK}} \hat{P}_R^2 \hat{P}^2 + h_7^{2\mathrm{PK}} \hat{P}_R^4 \right) + \frac{1}{\hat{R}^2} \left(h_8^{2\mathrm{PK}} \hat{P}^2 + h_9^{2\mathrm{PK}} \hat{P}_R^2 \right) + \frac{h_{10}^{2\mathrm{PK}}}{\hat{R}^3}$$

$$\mu \equiv \frac{m_A^0 m_B^0}{M} , \qquad M \equiv m_A^0 + m_B^0$$

The 17 $h_i^{N\mathrm{PK}}$ coefficients are computed explicitly and depend on :

- the 14 f_i (coordinate system) parameters
- the 8 fundamental parameters built from $m_A(\varphi)$ and $m_B(\varphi)$

II) A TEST PARTICLE IN A SSS EFFECTIVE METRIC

The effective Hamiltonian H_e

Geodesic motion in a static, spherically symmetric metric

In Schwarzschild-Droste coordinates (equatorial plane $\theta = \pi/2$) :

$$ds_e^2 = -A(r)dt^2 + B(r)dr^2 + r^2 d\phi^2$$

$A(r)$ and $B(r)$ are arbitrary.

Effective Hamiltonian $H_e(q,p)$:

$$H_e(q,p) = \sqrt{A\left(\mu^2 + \frac{p_r^2}{B} + \frac{p_\phi^2}{\hat{r}^2}\right)} \quad \text{with} \quad p_r \equiv \frac{\partial L_e}{\partial \dot{r}} \quad , \quad p_\phi \equiv \frac{\partial L_e}{\partial \dot{\phi}}$$

Can be **expanded** :

$$A(r) = 1 + \frac{a_1}{r} + \frac{a_2}{r^2} + \frac{a_3}{r^3} + \cdots$$
$$B(r) = 1 + \frac{b_1}{r} + \frac{b_2}{r^2} + \cdots$$

i.e. depends on **5 effective parameters** at 2PK order, to be determined.

Recap

• On one side, the 2PK two-body Hamiltonian $H(Q, P)$, depending on 17 parameters h_i^{NPK}

• On the other side, a simple effective Hamiltonian $H_e(q, p)$, depending on 5 parameters a_i, b_i.

Can we build a map between both Hamiltonians ?

EOB mapping : [Buonanno, Damour 98]

Requires imposing a functional relation $H_e = f_{\mathrm{EOB}}(H)$ by means of a canonical transformation

III) THE EOB MAPPING

1) Exploit the power of canonical transformations :

$$H(Q,P) \rightarrow H(q,p)$$

We take as a generic ansatz $G(Q,p)$ that depends on 9 parameters at 2PK order :

$$G(Q,p) = R\, p_r \left[\left(\alpha_1 \mathcal{P}^2 + \beta_1 \hat{p}_r^2 + \frac{\gamma_1}{\hat{R}} \right) + \left(\alpha_2 \mathcal{P}^4 + \beta_2 \mathcal{P}^2 \hat{p}_r^2 + \gamma_2 \hat{p}_r^4 + \delta_2 \frac{\mathcal{P}^2}{\hat{R}} + \epsilon_2 \frac{\hat{p}_r^2}{\hat{R}} + \frac{\eta_2}{\hat{R}^2} \right) + \cdots \right]$$

$$\mu = \frac{m_A^0 m_B^0}{m_A^0 + m_B^0}\ , \quad M = m_A^0 + m_B^0$$

$$r(Q,p) = R + \frac{\partial G}{\partial p_r}\ , \quad \phi(Q,p) = \Phi + \frac{\partial G}{\partial p_\phi}\ , \quad P_R(Q,p) = p_r + \frac{\partial G}{\partial R}\ , \quad P_\Phi(Q,p) = p_\phi + \frac{\partial G}{\partial \Phi}$$

- does not depend on time (conservative), nor on Φ (isotropic)
- generates 1PK and higher order coordinate changes

2) Relate H to H_e through a functional relation $H_e = f_{\rm EOB}(H)$

The exact quadratic relation

As proven recently to all orders from PM [Damour 2016]:

$$\frac{H_e(q,p)}{\mu} - 1 = \left(\frac{H(q,p) - M}{\mu}\right)\left[1 + \frac{\nu}{2}\left(\frac{H(q,p)-M}{\mu}\right)\right]$$

where $$\nu = \frac{m_A^0 m_B^0}{(m_A^0 + m_B^0)^2}\ , \qquad M = m_A^0 + m_B^0\ , \qquad \mu = \frac{m_A^0 m_B^0}{M}$$

- In Scalar-Tensor theories, yields a **unique** solution for H_e that does not depend on the f_i parameters (covariance).

- H_e contains all the 2PK physical information contained in $H(Q,P)$.

$$ds_e^2 = -A(r)dt + B(r)dr^2 + r^2 d\theta^2$$

Scalar-Tensor effective metric

$$A(r) = 1 - 2\left(\frac{G_{AB}M}{r}\right) + 2\left[\langle\bar{\beta}\rangle - \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \left[2\nu + \delta a_3^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^3 + \cdots$$

$$B(r) = 1 + 2\left[1 + \bar{\gamma}_{AB}\right]\left(\frac{G_{AB}M}{r}\right) + \left[2(2-3\nu) + \delta b_2^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \cdots$$

Consistent with General Relativity : $m_A(\varphi) = cst$ yields back

General Relativity 2PN effective metric [Buonanno, Damour 98]

$$A_{\mathrm{GR}}(r) = 1 - 2\left(\frac{G_*M}{r}\right) + 2\nu\left(\frac{G_*M}{r}\right)^3 + \cdots$$

$$B_{\mathrm{GR}}(r) = 1 + 2\left(\frac{G_*M}{r}\right) + 2(2-3\nu)\left(\frac{G_*M}{r}\right)^2 + \cdots$$

(*i*) The "bare" gravitational constant G_* is replaced by the effective one

$$G_* \to G_{AB} \equiv 1 + \alpha_A^0 \alpha_B^0$$

at all orders.

(*ii*) At 1PK level,

$$A(r) = 1 - 2\left(\frac{G_{AB} M}{r}\right) + 2\left[\langle\bar{\beta}\rangle - \bar{\gamma}_{AB}\right]\left(\frac{G_{AB} M}{r}\right)^2 + \cdots$$

$$B(r) = 1 + 2\left[1 + \bar{\gamma}_{AB}\right]\left(\frac{G_{AB} M}{r}\right) + \cdots$$

one recognizes the **PPN Eddington metric** written in Droste coordinates, with :

$$\beta^{\mathrm{Edd}} = 1 + \langle\bar{\beta}\rangle \ , \quad \gamma^{\mathrm{Edd}} = 1 + \bar{\gamma}_{AB}$$

Where

$$\langle\bar{\beta}\rangle \equiv \frac{m_A^0 \bar{\beta}_B^0 + m_B^0 \bar{\beta}_A^0}{m_A^0 + m_B^0} \qquad \bar{\gamma}_{AB} \equiv -\frac{2\alpha_A^0 \alpha_B^0}{1 + \alpha_A^0 \alpha_B^0} \qquad \bar{\beta}_A \equiv \frac{1}{2}\frac{\beta_A^0 (\alpha_B^0)^2}{(1 + \alpha_A^0 \alpha_B^0)^2}$$

Scalar-Tensor effective metric

$$A(r) = 1 - 2\left(\frac{G_{AB}M}{r}\right) + 2\left[\langle\bar\beta\rangle - \bar\gamma_{AB}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \left[2\nu + \delta a_3^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^3 + \cdots$$

$$B(r) = 1 + 2\left[1 + \bar\gamma_{AB}\right]\left(\frac{G_{AB}M}{r}\right) + \left[2(2-3\nu) + \delta b_2^{\mathrm{ST}}\right]\left(\frac{G_{AB}M}{r}\right)^2 + \cdots$$

(iii) 2PK corrections

$$\delta a_3^{\mathrm{ST}} \equiv \frac{1}{12}\Big[-20\bar\gamma_{AB} - 35\bar\gamma_{AB}^2 - 24\langle\bar\beta\rangle(1-2\bar\gamma_{AB}) + 4\big(\langle\delta\rangle - \langle\epsilon\rangle\big) + \nu\Big(-36(\bar\beta_A + \bar\beta_B) + 4\bar\gamma_{AB}(10+\bar\gamma_{AB}) + 4(\epsilon_A+\epsilon_B) + 8(\delta_A+\delta_B) - 24\zeta\Big)\Big]$$

$$\delta b_2^{\mathrm{ST}} \equiv \left[4\langle\bar\beta\rangle - \langle\delta\rangle + \bar\gamma_{AB}(9 + \frac{19}{4}\bar\gamma_{AB}) + \nu\left(2\langle\bar\beta\rangle - 4\bar\gamma_{AB}\right)\right]$$

$$\delta_A \equiv \frac{(\alpha_A^0)^2}{(1+\alpha_A^0\alpha_B^0)^2} \qquad \epsilon_A \equiv \frac{(\beta_A'\alpha_B^3)^0}{(1+\alpha_A^0\alpha_B^0)^3} \qquad \zeta \equiv \frac{\beta_A^0\alpha_A^0\alpha_B^0\beta_B^0}{(1+\alpha_A^0\alpha_B^0)^3}$$

IV) ST – PARAMETRISED EOB DYNAMICS

• The inversion of the $H_e = f_{\rm EOB}(H)$ hence defines a unique, "resummed" EOB Hamiltonian :

$$H_{\rm EOB} = M\sqrt{1 + 2\nu\left(\frac{H_e}{\mu} - 1\right)} \quad \text{where} \quad H_e = \sqrt{A\left(\mu^2 + \frac{p_r^2}{B} + \frac{p_\phi^2}{r^2}\right)}$$

The dynamics deduced from $H_{\rm EOB}$ and the "real" Hamiltonians H are, by construction, equivalent up to 2PK order.

• $H_{\rm EOB}$ hence defines a resummed dynamics, that may capture some features of the strong field regime.

A typical strong-field feature : orbital frequency at the ISCO

$$\Omega = \frac{\partial H_{\rm EOB}}{\partial H_e}\frac{\partial H_e}{\partial p_\phi} = \frac{ju^2 A}{G_{AB} M E\sqrt{1+2\nu(E-1)}} \qquad u = \frac{G_{AB}M}{r}$$

where, for circular orbits

$$j^2(u) = -\frac{A'}{(Au^2)'}\ , \quad E(u) = A\sqrt{\frac{2u}{(Au^2)'}}$$

The ISCO location, $u_{\rm ISCO}$, is such that :

$$\frac{A''}{A'} = \frac{(Au^2)''}{(Au^2)'}$$

→ **ST corrections to the ISCO location and frequency ?**

Last ingredient : the ST-corrected $A(u\,;\nu)$

$$u \equiv \frac{G_{AB}M}{r} \ , \quad \nu = \frac{m_A^0 m_B^0}{(m_A^0 + m_B^0)^2}$$

ST-corrected $A(u\,;\nu)$

$$A(u\,;\nu) = A^{\mathrm{GR}}_{\mathrm{2PN}}(u\,;\nu) + 2\epsilon_{\mathrm{1PK}}u^2 + (\epsilon^0_{\mathrm{2PK}} + \nu\epsilon^{\nu}_{\mathrm{2PK}})u^3$$

where

$$\epsilon_{\mathrm{1PK}} \equiv \langle\bar\beta\rangle - \bar\gamma_{AB}$$

$$\epsilon^0_{\mathrm{2PK}} \equiv \frac{1}{12}\Big[-20\bar\gamma_{AB} - 35\bar\gamma^2_{AB} - 24\langle\bar\beta\rangle(1-2\bar\gamma_{AB}) + 4\big(\langle\delta\rangle - \langle\epsilon\rangle\big)\Big]$$

$$\epsilon^{\nu}_{\mathrm{2PK}} \equiv -3(\bar\beta_A + \bar\beta_B) + \frac{1}{3}\bar\gamma_{AB}(10+\bar\gamma_{AB}) + \frac{1}{3}(\epsilon_A + \epsilon_B) + \frac{2}{3}(\delta_A + \delta_B) - 2\zeta$$

ST-Corrections described by 3 parameters, $(\epsilon_{\mathrm{1PK}}, \epsilon^0_{\mathrm{2PK}}, \epsilon^{\nu}_{\mathrm{2PK}})$

- **BUT** numerically driven by $(\alpha_A^0)^2$ (c.f. DEF, diagrammatic methods)
- When $(\alpha_A^0)^2 << 1$, $\epsilon_{\mathrm{1PK}} \sim \epsilon^0_{\mathrm{2PK}} \sim \epsilon^{\nu}_{\mathrm{2PK}}$ and ST-corrections are perturbative

ST corrections to the strong-field regime

In this perturbative approach, **best available EOB-NR function** for GR :

$$A^{\mathrm{GR}}_{\mathrm{2PN}}(u\,;\nu) \rightarrow \boxed{A^{\mathrm{GR}}_{\mathrm{EOBNR}}(u\,;\nu) = \mathcal{P}^1_5[A^{Taylor}_{\mathrm{5PN}}]}$$

i.e. the $(1,5)$ Padé approximant of the truncated 5PN expansion :

$$A^{Taylor}_{\mathrm{5PN}} = 1 - 2u + 2\nu u^3 + \nu a_4 u^4 + (a_5^c + a_5^{\ln} \ln u)u^5 + \nu(a_6^c + a_6^{\ln} \ln u)u^6$$

[Damour, Nagar, Reisswig, Pollney 2016]

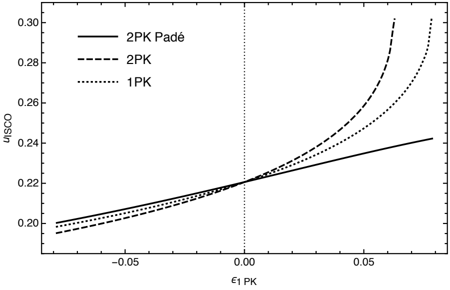
- smoothly connected to Schwarzschild when $\nu \rightarrow 0$
- $a_6^c(\nu)$ is obtained by calibration with Numerical Relativity

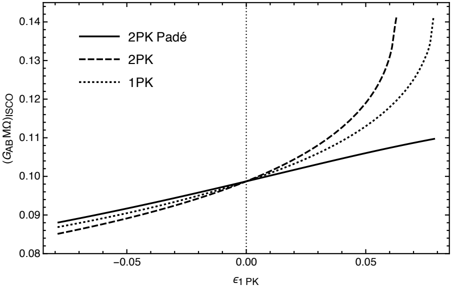
ISCO Locations and frequency, equal-mass case ($\nu = 1/4$)

- 2PK Padeed corrections,

$$A = \mathcal{P}_5^1[A^{\rm GR}_{\rm EOBNR}(u\,;\nu) + 2\epsilon_{\rm 1PK}u^2 + (\epsilon^0_{\rm 2PK} + \nu\epsilon^\nu_{\rm 2PK})u^3]$$

$u_{\rm ISCO}(\epsilon_{\rm 1PK})$ $\qquad$ $G_{AB}M\Omega_{\rm ISCO}(\epsilon_{\rm 1PK})$





$$\left.\frac{d(G_{AB}M\Omega)_{ISCO}}{d\epsilon_{\rm 1PK}}\right|_{\nu=1/4} \simeq 0.13$$

relative correction to GR significant ($\sim 10\%$) when $\epsilon_{\rm 1PK} \sim 10^{-2} - 10^{-1}$

Concluding remarks :

• Binary pulsar experiments have put **stringent constraints on ST theories** (no dipolar radiation)

$$(\alpha_A^0)^2 < 4 \times 10^{-6}$$

For **any** body A, regardless of its EOS or self-gravity.

• The ISCO ST-correction (significant for $(\alpha_A^0)^2 \gtrsim 10^{-2}$) seems unlikely to improve binary pulsar constraints.

However :

• The interferometers LIGO-Virgo or even LISA are designed to detect highly redshifted sources. Cosmological history of ST theories ?