Quantum decoherence from stochastic backgrounds of gravitational waves

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Introduction

- Quantum mechanics assume flat spacetime (Minkowski).

- But spacetime is curved:
  - in particular, stochastic backgrounds of gravitational waves.
  - can those backgrounds affect quantum properties of systems? How? At which scales?

- Quantum decoherence: a quantum system interacting with an environment evolves towards a classical behavior.

- Gravitational waves backgrounds define a universal environment (everything is coupled to gravitation through its energy momentum tensor).
**Gravitational decoherence**

- **Feynman Lectures on Gravitation (1962)**
  
  → Interaction with our gravitational environment could be at the origin of the classical behavior of macroscopic systems.

  → Planck scales argument.

  \[
  t_P = \sqrt{\frac{\hbar G}{c^5}} \approx 5 \times 10^{-44} \text{s} \quad \ell_P = \sqrt{\frac{\hbar G}{c^3}} \approx 10^{-35} \text{m}
  \]

  → But \( m_P = \sqrt{\frac{\hbar c}{G}} \approx 22 \mu \text{g} \) lies between microscopic and macroscopic masses.

  → Is it accidental or is it a hint that space-time fluctuations are at the origin of some universal decoherence mechanism?

- The Planck scale transition can be seen by comparing the Compton length \( \ell_C = \frac{\hbar}{m c} \) to the Planck length \( \ell_P = \frac{\hbar}{m_P c} \)

  \[
  m < m_P \iff \ell_C > \ell_P \quad m > m_P \iff \ell_C < \ell_P
  \]
From quantum/classical border...

- **Microscopic objects**
  - electrons, photons, atoms, molecules...
  - \( C_{60} \)
  - interferences

- **Macroscopic objects**
  - soccer ball, cats, planetary systems.
  - No interferences

- Where is the borderline (in term of mass ? size ? etc...) ?

(from Zurek’s original picture)
... to new gedanken experiments

• Would it be possible to prove the existence of intrinsic fluctuations of space-time by observing some universal diffusion?
  → situation analogous to that which allowed physicists to prove the existence of atoms by observing Brownian diffusion in the beginning of the 20th century.


• Is this feasible with present state-of-the-art instruments such as atomic or optical interferometers?
  → Spatial projects: HYPER, MWXG, GAUGE, QUEST...

Decoherence in HYPER, an artist’s view (ESA, 2000)
Gravitational environment

- General relativity is the effective theory of gravity at experimentally accessible frequencies
  → the associated intrinsic fluctuations are known, they are the gravitational waves.

- Gravitational waves (GW) are freely propagating solutions of linearized Einstein equations.
  → perturbation of the metric: $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, $h_{\mu\nu} \ll 1$
  → TT gauge: only $h_{ij}$

- Coupling to gravitation: $\mathcal{L}_{S/\mathcal{E}} = \frac{1}{2} h_{\mu\nu} T^{\mu\nu}$
  → reduces to a quadrupole coupling for non relativistic motion $\mathcal{L}_{S/\mathcal{E}} = \frac{1}{4} h_{ij}(t) \tilde{Q}^{ij}(t)$
  → analog to dipole approximation of electromagnetism.

- GW backgrounds → collection of harmonic oscillators that can be quantized ($\hat{h}_{ij}$) even if we don’t have a full theory of quantum gravity.
Gravitational waves backgrounds

- Classical metric perturbations are characterized by a noise spectrum $S_h[\omega]$

  $$\langle h_{ij}(t)h_{kl}(0) \rangle = \delta_{ijkl} \int \frac{d\omega}{2\pi} S_h[\omega]e^{-i\omega t}$$

- Described also by a noise temperature or a graviton number

  $$S_h[\omega] = \frac{16G}{5c^5} k_B T_{gw}[\omega] = \frac{16G}{5c^5} n_{gw}[\omega] \hbar \omega$$

- They correspond to an enormous temperature but an extremely weakly coupled environment
  
  $\rightarrow$ $T_{gw}$ is certainly not an equilibrium temperature ! (no blackbody for gravitational radiation)

  $\rightarrow$ GW detectors need $n_{gw} \sim 10^{31}$ gravitons at 1kHz

- It is an ideal environment to create large decoherence without dissipation.
Galactic backgrounds

Binary confusion background, generated by all unresolved binary systems in our Galaxy and its vicinity

- Classical environment (stochastic nature comes from misknowledge of the sources)
- A nearly constant spectrum between $1\mu$Hz and $1$ mHz
  \[ \sqrt{S_h} \sim 10^{-17} \text{Hz}^{-\frac{1}{2}} \]
- An enormous noise temperature
  \[ T_{gw} \sim 10^{41} \text{K} \]
- An extremely weakly coupled reservoir (fortunately !)

B. Schutz *Class. Quant. Grav. 1999, gr-qc/9911034*
Cosmic backgrounds

Relic background with a primordial origin

\[ S_h = \frac{6\pi H_0^2 \Omega_{gw}}{\omega^3} \]

\[ \Omega_{gw} = 10^{-14} \]

→ quantum environment (highly squeezed vacuum stemming from amplification of initial vacuum fluctuations)

→ \( \Omega_{gw} \) energy density of GW compared to the critical density

→ \( H_0 \) Hubble constant

→ in some simple models, \( \Omega_{gw} \) is a constant (unfortunately unknown)

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Quantum decoherence

- Path integral interpretation: the quantum evolution is governed by interferences between quantum paths.
- Quantum coherences are characterized by the contrast of interferences between those quantum paths.

\[ I = |A_+ + A_-|^2 = |A_+|^2 + |A_-|^2 + 2\text{Re}(A_+A_-^*) \]

- In presence of an environment \( \mathcal{E} \)

\[ I = |A_+|^2 + |A_-|^2 + 2\text{Re}(A_+A_-^* \mathcal{F}[q_+,q_-]) \]

\[ \langle \mathcal{E}_+|\mathcal{E}_+ \rangle = \langle \mathcal{E}_-|\mathcal{E}_- \rangle = 1 \quad \mathcal{F}[q_+,q_-] = \langle \mathcal{E}_+|\mathcal{E}_- \rangle \]

- \( \mathcal{F}[q_+(t),q_-(t)] \): Feynman-Vernon influence functional.
  → it gives the coherence between two paths \( q_+(t) \) and \( q_-(t) \).

\[ \mathcal{V} = |\mathcal{F}[q_+,q_-]| = |\langle \mathcal{E}_+|\mathcal{E}_- \rangle| \]

other approaches developed in R. Penrose 1996, Kok & Yurtsever 2003
Quantum decoherence

- The influence functional can be written as an exponential of linear and two quadratic kernels:

\[
\mathcal{F}[q_+, q_-] = \exp \left( i \Delta \varphi_{\text{lin}} - \frac{1}{2} \Delta \varphi_{\text{noise}}^2 - \frac{i}{2} \Delta \varphi_{\text{diss}}^2 \right)
\]

- Double limit of weak coupling and high temperature:

\[\rightarrow \Delta \varphi_{\text{diss}}^2 \text{ negligible.}\]

- Linear term

\[\Delta \varphi_{\text{lin}} = \frac{1}{4\hbar} \int_{t_i}^{t_f} (\bar{Q}^{ij}[q_+(t)] - \bar{Q}^{ij}[q_-(t)])\langle \hat{h}_{ij}(t) \rangle dt\]

- Noise kernel

\[\Delta \varphi_{\text{noise}}^2 = -\frac{1}{4\hbar} \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} ds \langle \bar{Q}^{ij}[q_+(t)] - \bar{Q}^{ij}[q_-(t)] \rangle \sigma_{ijkl}(t-s) (\bar{Q}^{kl}[q_+(s)] - \bar{Q}^{kl}[q_-(s)])\]

\[\sigma_{ijkl}(\tau) = \frac{1}{4\hbar} \left( \langle \hat{h}_{ij}(\tau) \cdot \hat{h}_{kl}(0) \rangle - \langle \hat{h}_{ij}(\tau) \rangle \langle \hat{h}_{kl}(0) \rangle \right)\]

\[\rightarrow \text{the galactic background does not contribute in this kernel.}\]
Application to atomic interferometers

- HYPER project: the lasers used for stimulated Raman transition provide nearly freely falling mirrors and beam splitters for atoms.
  → sensitive to GW.

- Linear term
  \[ \Delta \varphi_{\text{lin}} = \frac{1}{2\hbar} \int h_{\mu\nu} \frac{P^\mu P^\nu}{m} dt \]

- Noise term
  \[ \Delta \varphi_{\text{noise}}^2 = \int \frac{d\omega}{2\pi} A[\omega] S_h[\omega] \]

- Simplified white noise model
  \[ \Delta \varphi_{\text{noise}}^2 \sim \left( \frac{\Delta E_k}{\hbar} \right)^2 S_h \tau \]
  → Brownian diffusion of the phase.
  → \( \Delta E_k \) variation of kinetic energy (in comoving frame) by the beam splitter.

http://sci.esa.int/hyper
Discussion

- Relevant parameters for the estimation of decoherence:
  - kinetic energy (in comoving frame) of the probe (not its mass energy!)
  - size of the interferometer (and separation angle)
  - level of the noise spectrum ($S_h$)

- All existing interferometers verify $\Delta \varphi_{\text{noise}}^2 \ll 1$
  - In HYPER, $\Delta \varphi_{\text{noise}}^2 \sim 10^{-12}$

- In the mean time, gravitational decoherence is the dominant mechanism for large macroscopic systems (planetary motions)


$$t_{\text{dec}} \sim \frac{\hbar}{MR\Delta x \sqrt{\langle R_{0i0j} R_{0i0j} \rangle}} \sim 5 \times 10^{-51} \text{s} \left( \frac{10^{-14}}{\Omega_{gw}} \right)^{1/2} \left( \frac{1 \text{ m}}{\Delta x} \right)$$

- Challenge: would it be possible to design experiments allowing one to explore the transition on « mesoscopic » objects?

link to Feynman argument on the Planck mass
« Challenge » for matter wave interferometry


\[ E \sim 5 \text{ keV} \quad \sin(2\alpha) \sim 1 \quad A \sim 1 \text{ m}^2 \]

Freely falling beam splitters

Multi shell fullerene in supersonic beam
(~ 3000 carbon atoms).

GPhyS Kick Off meeting, 20-22 octobre 2009, les Houches
State of the art

→ Interferences with large molecules.


→ Nevertheless, small separation angle, small kinetic energy and not freely falling beam splitters.

\[ E \sim 100 \text{ meV} \quad \Delta \varphi_{\text{noise}}^2 \ll 1 \]
Optical interferometer

LISA

http://sci.esa.int/home/lisa/
http://lisa.jpl.nasa.gov/

\[ \Delta \varphi_{\text{Noise}} \ll 1 \]

LISA is microscopic!

Good news for the fringes…
EPR experiments on large distances

- Are there limits on the distance between two entangled quantum systems?


http://www.quantum.at/quest
EPR in presence of a GW

- Quantum correlation function
  \[ E(\varepsilon^A, \varepsilon^B) = C(\varepsilon^A, \varepsilon^B) + C(\varepsilon^A, \varepsilon^B) - C(\varepsilon^A, \varepsilon^B) - C(\varepsilon^A, \varepsilon^B) \]

- Rotation of polarisation
  \[ \alpha_{\pm}(t) = \frac{1}{2} \int_{ct}^{ct+cT_\pm} (\partial_1 h_{23} - \partial_2 h_{13}) d\sigma \]

- An EPR experiment can be viewed as an interferometric signal
  \[ E(\varepsilon^A, \varepsilon^B)(t) = -\cos[2(\Theta - \alpha(t))] \]

\[ \alpha = \alpha_{\rightarrow} - \alpha_{\leftarrow} \]
EPR in stochastic GW backgrounds

- Quantum correlation function
  \[ E(\epsilon^A, \epsilon^B) = -\langle \cos[2(\Theta - \alpha)] \rangle = \cos(2\Theta - 2\langle \alpha \rangle) e^{-2\Delta \alpha^2} \]

- Decrease of the CHSH Bell test parameter
  \[ S \leq 2\sqrt{2} \exp (-2\Delta \alpha^2) \]

- At the largest cosmological scales, decoherence is dictated by the value of the energy density of GW
  \[ \Delta \alpha^2 \sim \Omega_{GW} \]
Conclusions

- Gravitational decoherence sets a natural border between classical and quantum objects (Feynman intuition was correct).

- Experimental challenge to monitor decoherence (new ideas ? BEC ? Quantum superposition of micro mechanical mirrors ? etc...).

- Quantum correlation based on polarisation can survive over the largest cosmological scales.
The influence functional can be written as an exponential of linear and two quadratic kernels:

\[
\mathcal{F}[q_+, q_-] = \exp \left( \frac{i}{4\hbar} \int_{t_i}^{t_f} \left( \ddot{Q}^{ij}[q_+(t)] - \ddot{Q}^{ij}[q_-(t)] \right) \langle \hat{h}_{ij}(t) \rangle \, dt \right) 
\times \exp \left( -\frac{1}{4\hbar} \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} ds \left( \ddot{Q}^{ij}[q_+(t)] - \ddot{Q}^{ij}[q_-(t)] \right) \sigma_{ijkl}(t-s) \left( \ddot{Q}^{kl}[q_+(s)] - \ddot{Q}^{kl}[q_-(s)] \right) \right) 
\times \exp \left( -\frac{1}{4\hbar} \int_{t_i}^{t_f} dt \int_{t_i}^{t_f} ds \left( \ddot{Q}^{ij}[q_+(t)] + \ddot{Q}^{ij}[q_-(t)] \right) \xi_{ijkl}(t-s) \left( \ddot{Q}^{kl}[q_+(s)] - \ddot{Q}^{kl}[q_-(s)] \right) \right)
\]

- Noise kernel.
- Dissipation kernel (imaginary) (spontaneous emission of GW).

\[
\sigma_{ijkl}(\tau) = \frac{1}{4\hbar} \left( \langle \hat{h}_{ij}(\tau) \cdot \hat{h}_{kl}(0) \rangle - \langle \hat{h}_{ij}(\tau) \rangle \langle \hat{h}_{kl}(0) \rangle \right)
\]

\[
\xi_{ijkl} = \frac{1}{4\hbar} \langle [\hat{h}_{ij}(\tau), \hat{h}_{kl}(0)] \rangle = i\delta_{ijkl} \int_0^\infty d\omega \frac{2G\omega}{5\pi c^5} \sin(\omega \tau)
\]