Numerical models for superfluid neutron stars & application to pulsar glitches

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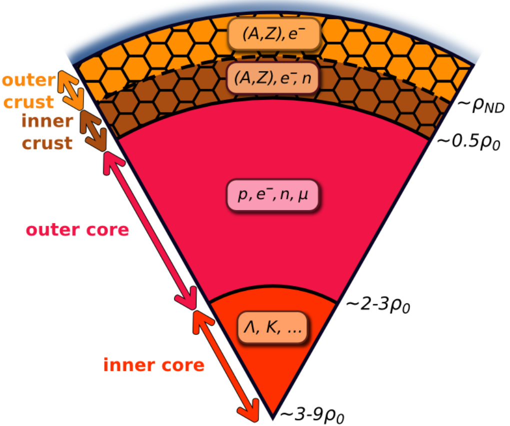
Superfluidity in neutron stars



[adapted from Langlois, "Superfluidity in relativistic neutron stars", 2002 & Haensel, Potekhin & Yakovlev, "Neutron stars 1 : Equation of state and structure", 2007.]

Theoretical considerations

[Baym, Pethick & Pines, *Nature*, 1969 & Pines & Alpar, *Nature*, 1985]

$$T \lesssim T_c \sim 10^9 - 10^{10}\ \mathrm{K}$$

- **superfluid** neutrons in the core & in the inner crust,
- **superconducting** protons in the core.

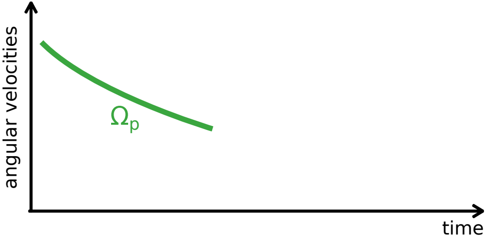
Two-fluid model

Consequence of superfluidity:

several **dynamically distinct** components inside neutron stars.

Two fluids

- Charged particles:
  $\Omega_p = \Omega$ ($\to$ pulsar)



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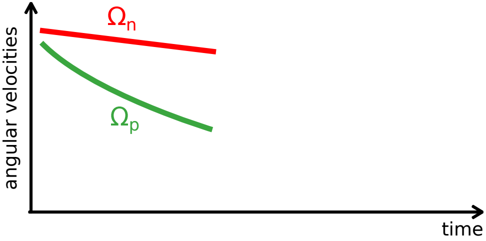
- Superfluid neutrons: $\Omega_n \gtrsim \Omega_p$



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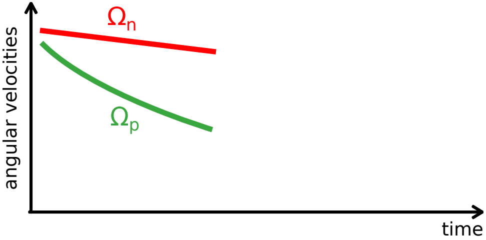

Couplings:

- **Entrainment** (non dissipative)
- **Mutual friction** (dissipative)

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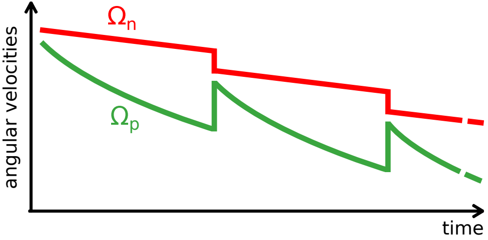


Couplings:

- **Entrainment** (non dissipative)
- **Mutual friction** (dissipative) $\rightarrow$ glitches

Evidence for superfluidity
[Anderson & Itoh, *Nature*, 1975]

Long relaxation time scales observed in *pulsar glitches*.

Purposes of the present work

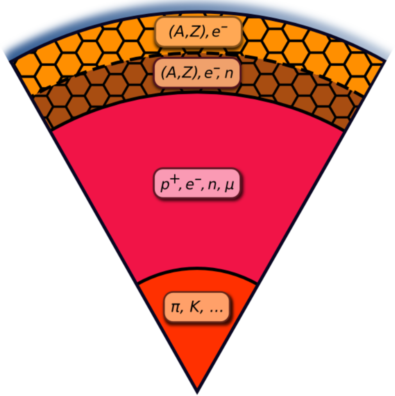
Previous works:

- *Theoretical* model with two fluids in GR developed by **Carter, Langlois, *et al.*** (1990s).
- Prix, Novak & Comer, *PRD*, 2005: *Numerical* model for stationary superfluid neutron stars implemented in LORENE.
  ⇢ polytropic EoS (non realistic).

Purposes

- Compute equilibrium configurations of rotating superfluid neutron stars considering 2-fluid EoSs based on microphysics,
- Give a simple numerical model concerning pulsar glitches.

Basic assumptions



Equilibrium configurations

- isolated star,
- $T = 0$,
- no magnetic field,
- dissipative effects are neglected,
- uniform composition $\rightarrow p, e^-, n$,
- asymptotically flat, stationary, axisymmetric & circular metric,
- rigid-body rotation: Ω_n, Ω_p.

System = two **perfect** fluids coupled by *entrainment*:

- superfluid neutrons $\rightarrow \vec{n}_n = n_n \vec{u}_n$,
- protons & electrons $\rightarrow \vec{n}_p = n_p \vec{u}_p$.

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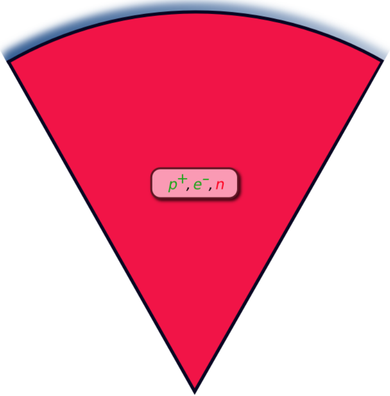

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Canonical two-fluid hydrodynamics

[Carter, "Covariant theory of conductivity in ideal fluid or solid media", 1989, Comer & Langlois, *CQG*, 1994, Carter & Langlois, *PRD*, 1995 & Carter & Langlois, *Nuc. Phys. B*, 1998.]

Energy-momentum tensor

$$T_{\alpha\beta} = n_{\mathrm{n}\alpha} p^{\mathrm{n}}_{\beta} + n_{\mathrm{p}\alpha} p^{\mathrm{p}}_{\beta} + \Psi g_{\alpha\beta}$$

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Entrainment matrix:

$$\begin{cases} p^{\mathrm{n}}_{\alpha} & = \mathcal{K}^{\mathrm{nn}} n^{\mathrm{n}}_{\alpha} + \mathcal{K}^{\mathrm{np}} n^{\mathrm{p}}_{\alpha} \\ p^{\mathrm{p}}_{\alpha} & = \mathcal{K}^{\mathrm{pn}} n^{\mathrm{n}}_{\alpha} + \mathcal{K}^{\mathrm{pp}} n^{\mathrm{p}}_{\alpha} \end{cases}$$

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$\Rightarrow$ entrainment effect

without entrainment

Equation of State

$$\mathcal{E}(n_\mathrm{n}, n_\mathrm{p}, \Delta^2) \leftrightarrow \Psi(\mu^\mathrm{n}, \mu^\mathrm{p}, \Delta^2)$$

First law of thermodynamics (T=0)

$$\mathrm{d}\mathcal{E} = \mu^\mathrm{n}\,\mathrm{d}n_\mathrm{n} + \mu^\mathrm{p}\,\mathrm{d}n_\mathrm{p} + \alpha\,\mathrm{d}\Delta^2$$

Relativistic Mean-Field Theory:

nucleon-nucleon interactions ⇔ exchange of effective mesons

$$\mathcal{L} = \boxed{\mathcal{L}_b} + \boxed{\mathcal{L}_\sigma + \mathcal{L}_\omega + \mathcal{L}_\rho + \mathcal{L}_\delta} + \boxed{\mathcal{L}_\mathrm{int}}$$

free baryons — free mesons — interaction

EoSs:

- DDH (without δ meson)
- DDHδ (with δ meson)

⇢ adapted to a two-fluid system coupled by entrainment.

[Comer & Joynt, 2003 & Gusakov, Kantor & Haensel, 2009.]

Entrainment effects

Dynamical effective mass:

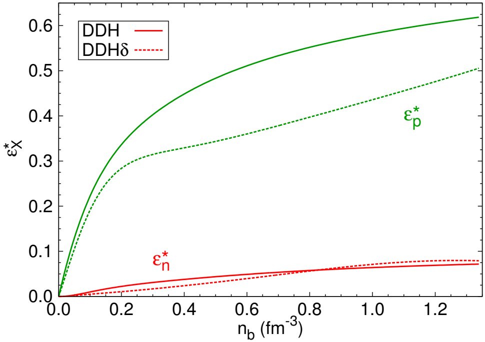
$$^3\vec{p}_X = m_X^* \; ^3\vec{u}_X$$

$\hookrightarrow$ in the *rest frame* of the second fluid.

Zero-*velocity* frame

$$m_X^* = \mu^X \left(1 - \varepsilon_X^*\right)$$

$\hookrightarrow \quad \varepsilon_X^* = \frac{2\alpha}{n_X \mu^X}$

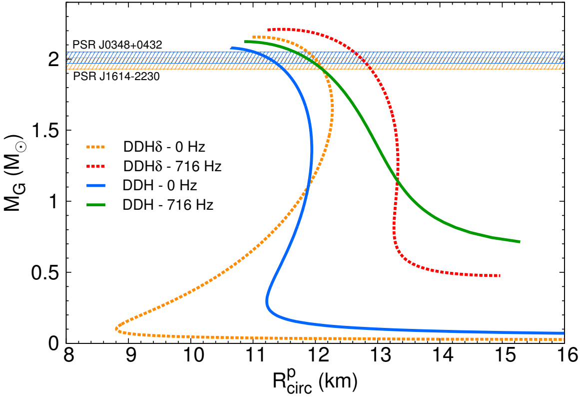


$\dashrightarrow$ assuming β-equilibrium & $\Delta^2 = 0$.

Global quantities

Surface definitions

$n_n = 0$ & $n_p = 0$



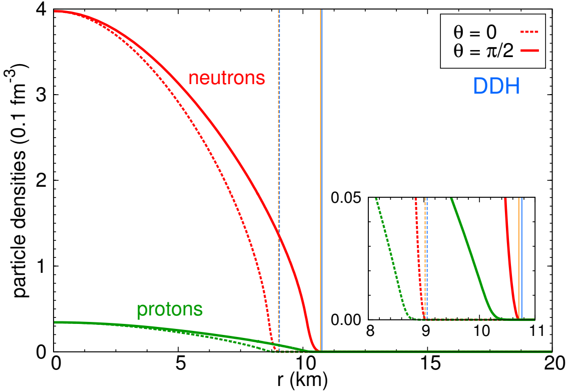
- Gravitational mass:
$$M_G = M^B + E_{bind},$$
- Circumferential radius:
$$R^X_{circ,\ eq} = \mathcal{C}^X / 2\pi.$$

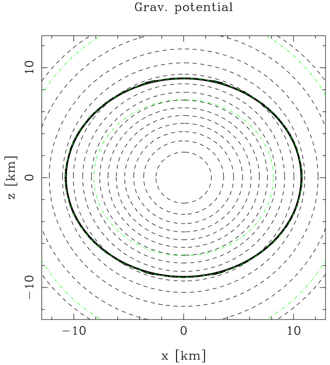
Virial identities:

$GRV \sim 10^{-7} - 10^{-5}$

Density profiles

$$M_G = 1.4\ \mathrm{M}_\odot,\ \Omega_\mathrm{n}/2\pi = \Omega_\mathrm{p}/2\pi = 716\ \mathrm{Hz}$$



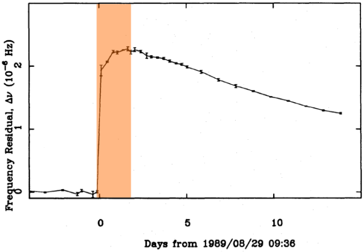


$\dashrightarrow\ n_b(0) \simeq 0.44\ \mathrm{fm}^{-3}\ \&\ x_p(0) \simeq 0.08.$

Modelling glitch rise

Bulk model for pulsar glitches:

$\Delta\Omega \gtrsim \Delta\Omega_c \Rightarrow$ angular momentum transfer through **mutual friction**



[Lyne, Pritchard & Smith, *MNRAS*, 1993.]

Typical time scales

- **rise time:**

$$\begin{cases} \tau_r \lesssim 40 \text{ s} - 2 \text{ min} & \textbf{Vela} \\ \tau_r \sim \text{a few hours} & \textbf{Crab} \end{cases}$$

[Dodson, McCulloch & Lewis, *ApJL*, 2002 & Lyne, Pritchard & Smith, *MNRAS*, 1993.]

- **dynamical time:**

$$\tau_{\text{dyn}} \simeq 0.1 \text{ ms}$$

[Epstein, *ApJ*, 1988 & Shapiro, *ApJ*, 2000.]

⇢ Series of equilibrium configurations with **constant** M^B and J.

Angular momentum transfer

Evolution equations:

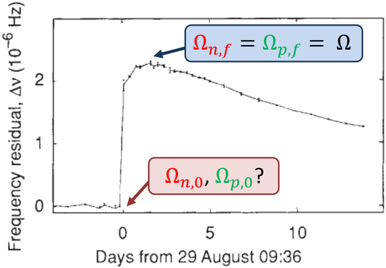
$$\begin{cases} \dot{J}_n = +\Gamma_{int}, \\ \dot{J}_p = -\Gamma_{int}. \end{cases}$$

Mutual friction moment

[Langlois, Sedrakian & Carter, *MNRAS*, 1998 & Sidery, Passamonti & Andersson, *MNRAS*, 2010.]

$$\Gamma_{int} = \mathcal{B}\,(\Omega_p - \Omega_n) \int \Gamma_n n_n \varpi_n h_\perp^2 \, d\Sigma$$

$\hookrightarrow$ Computation of $\Omega_n(t)$ & $\Omega_p(t)$ profiles from $\Omega_{n,0} > \Omega_{p,0}$



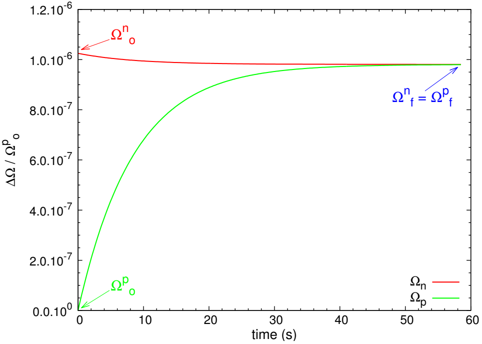
Initial lag = trigger threshold:

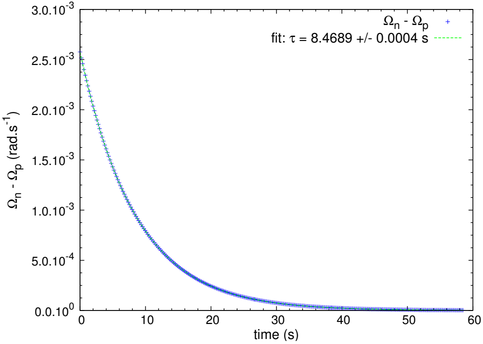
$$\Delta\Omega_c = \Omega_{n,0} - \Omega_{p,0} \simeq 10^{-9} - 10^{-5}\ \Omega$$

Preliminary results

Inputs:

$$\Delta\Omega/\Omega = 10^{-6},\ \Omega^{\rm n}_f/2\pi = \Omega^{\rm p}_f/2\pi = 400\ \text{Hz},\ M^B = 1.7\ \text{M}_\odot\ \&\ \mathcal{B} = 10^{-6}$$





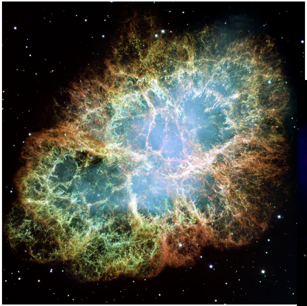
$$\Omega_{\rm n}(t) - \Omega_{\rm p}(t) \propto e^{-t/\tau_r} \Rightarrow \tau_r \sim 8.5\ \text{s}$$

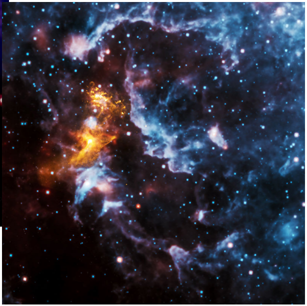
Conclusion

- Equilibrium configurations for superfluid neutron stars with realistic EoSs, using LORENE,
- Bulk model for pulsar glitches seen as angular momentum transfers through mutual friction force, in GR.

Future work:

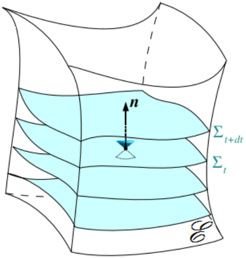
- τ_r as a function of $\mathcal{B}$, M^B, ... for different EoSs,
- Confrontation with accurate observations of glitches,
- Evolution in time of mass quadrupole $\mathcal{Q} \to$ GWs.





Thank you!

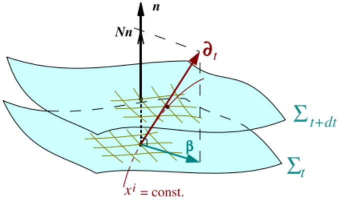
Formalisme 3+1



Feuilletage de l'espace-temps $(\mathcal{E}, \boldsymbol{g})$ par $(\Sigma_t)_{t\in\mathbb{R}}$, de normale (locale) unitaire $\vec{n}$

Observateurs euleriens $\mathcal{O}_n$: 4-vitesse $= \vec{n}$

- Fonction lapse N : $\vec{n} = -N\vec{\nabla}t$,
- Vecteur shift $\vec{\beta}$: $\vec{\partial}_t = N\vec{n} + \vec{\beta}$.



Métrique 3+1 :

$$g_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta = -N^2\,\mathrm{d}t^2 + \gamma_{ij}\left(\mathrm{d}x^i + \beta^i\,\mathrm{d}t\right)\left(\mathrm{d}x^j + \beta^j\,\mathrm{d}t\right)$$

Résolution numérique : méthode du point fixe

Paramètres d'entrée :

- une EOS
- H_c^n, H_c^p
- Ω_n, Ω_p

$i = 0$

Initialisation :

- $N = A = B = 1$ et $\omega = 0, \forall\,(r,\theta)$
- $U_n = U_p = 0$
- $H_0^i(r,\theta) = H_c^i\left(1 - \frac{r^2}{R^2}\right)$

Test de convergence

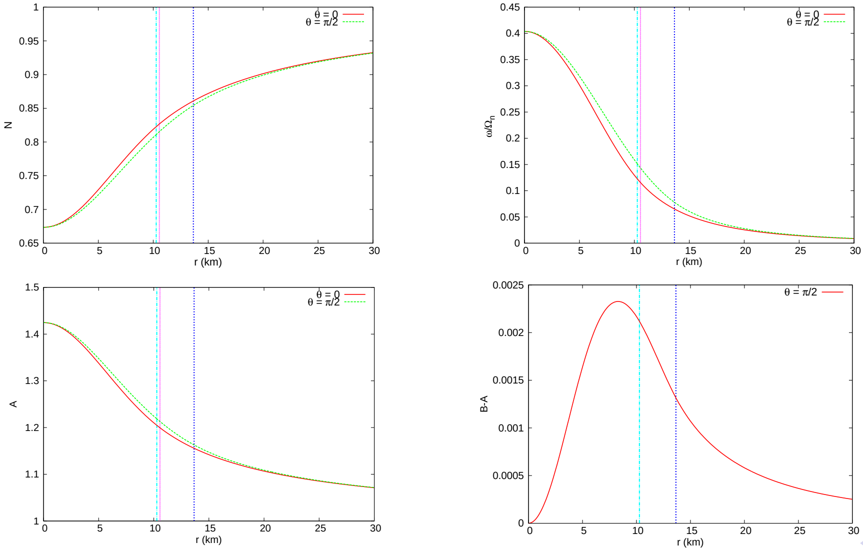
$$|H^i_{k+1}(r,\theta) - H^i_k(r,\theta)| < \epsilon$$

Étapes lors d'une itération

En chaque point $(\mu^{\mathrm{n}}, \mu^{\mathrm{p}}, \Delta^2)$, on calcule :

1. Ψ, n_{n}, n_{p} et α à partir de l'EOS
2. Les termes sources E, p_φ, $S^i{}_i$,
3. Résolution des équations d'Einstein,
4. Les termes cinétiques U_{i} et Γ_{i},
5. Calculs de H^i_{k+1}.

Résultats - coefficients métriques



Spacetime metric

[Bonazzola, Gourgoulhon, Salgado & Marck, *A&A*, 1993 & Prix, Novak & Comer, *PRD*, 2005.]

Rotating neutron stars, at **equilibrium**, described by $(\mathcal{E}, \boldsymbol{g})$:

- asymptotically flat: $\boldsymbol{g} \to \boldsymbol{\eta}$ at spatial infinity ($r \to +\infty$),
- stationary & axisymmetric: $\frac{\partial g_{\alpha\beta}}{\partial t} = \frac{\partial g_{\alpha\beta}}{\partial \varphi} = 0$,
- circular: perfect fluids $\Rightarrow$ *purely circular* motion around the rotation axis with Ω_{n}, Ω_{p} (+ **rigid rotation**).

Spacetime metric in quasi-isotropic coordinates:

$$g_{\alpha\beta}\,\mathrm{d}x^\alpha\,\mathrm{d}x^\beta = -N^2\,\mathrm{d}t^2 + A^2(\mathrm{d}r^2 + r^2\,\mathrm{d}\theta^2) + B^2 r^2 \sin^2\theta(\mathrm{d}\varphi - \omega\,\mathrm{d}t)^2$$

At spatial infinity

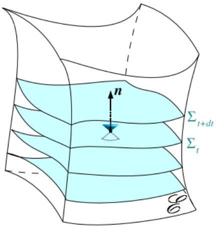
$$N, A, B \to 1 \quad \& \quad \omega \to 0$$

Angular momenta

Axisymmetry $\leftrightarrow$ $\vec{\chi}$

Komar definition:

$$J_{\rm K} = -\int_{\Sigma_t} \underbrace{\boldsymbol{T}(\vec{\boldsymbol{n}}, \vec{\chi})}_{-p_\varphi} \, \mathrm{d}^3 V$$



Eulerian observer $\vec{\boldsymbol{n}}$ (3+1)

Angular momentum of each fluid

[Langlois, Sedrakian & Carter, *MNRAS*, 1998.]

$$p_\varphi = \underbrace{\Gamma_{\rm n} n_{\rm n} p^{\rm n}_\varphi}_{j^{\rm n}_\varphi} + \underbrace{\Gamma_{\rm p} n_{\rm p} p^{\rm p}_\varphi}_{j^{\rm p}_\varphi}$$

$$J_X = \int_{\Sigma_t} j^X_\varphi \; A^2 B r^2 \sin\theta \, \mathrm{d}r \, \mathrm{d}\theta \, \mathrm{d}\varphi$$

Rk: $\Omega_X = 0 \not\Rightarrow J_X = 0$, if the second fluid is rotating!

Tabulated EoS