Backreaction of the infrared modes of scalar fields on de Sitter geometry

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Table of Contents

Motivations

Why de Sitter ?

- It is maximally symmetric
- It is relevant for inflation

For scalar field in dS,

- Large gravitational effects in the infrared (superhorizon scales)
- Infrared modes are amplified
- Interactions cannot be treated perturbatively

Table of Contents

Semiclassical approach

The theory is described by an **effective action** $\Gamma[\varphi, g]$, the Legendre transform of $\mathscr{W}[j, g]$ defined as

$$e^{i\mathscr{W}[j,g]} = \int \mathscr{D}\hat{\varphi} e^{iS[\hat{\varphi},g]+i\int j\hat{\varphi}}, \quad \Gamma[\varphi, g] = \mathscr{W}[j, g] - j \cdot \varphi$$

with $g_{\mu\nu}$ the background metric.
The action S will be typically an $O(N)$ theory with φ^4 interaction.

Non perturbative renormalization group

A. Kaya '13; M. Guilleux, J. Serreau '15

Add a regulator

$$i\Delta S_\kappa[\hat{\varphi}, g] = i\int_{x,y} R_\kappa(x,y)\hat{\varphi}(x)\hat{\varphi}(y).$$

And define an effective action which interpolates between S and Γ

$$\Gamma_\kappa[\varphi, g] = \mathscr{W}_\kappa[j, g] - j\cdot\varphi - \Delta S_\kappa[\varphi, g]$$

The **physical values** for g and φ are simultaneously determined at a scale κ through

$$\frac{\delta\Gamma_\kappa}{\delta\varphi} = 0, \quad \frac{\delta\Gamma_\kappa}{\delta g^{\mu\nu}} = 0$$

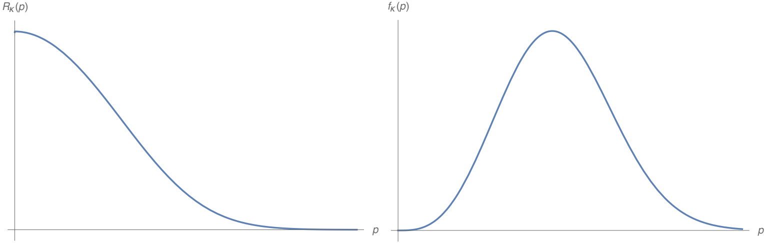
which we evaluate at constant values of φ.

Non perturbative renormalization group 2

We want to solve the flow of Γ_κ : it obeys the **Wetterich equation**

$$\dot{\Gamma}_\kappa = \frac{1}{2}\,\mathrm{tr}\,\dot{R}_\kappa(\Gamma^{(2)}_\kappa + R_\kappa)^{-1}.$$

This equation is regulated both in the infrared and the ultraviolet (f_κ is a typical integrand in the r.h.s.)



De Sitter Background

$$\frac{\delta\Gamma_\kappa}{\delta g^{\mu\nu}} = 0 \qquad \Rightarrow \qquad G^\kappa_{\mu\nu} = \left\langle T^\kappa_{\mu\nu} \right\rangle$$

- Without regulator, $\left\langle T_{\mu\nu} \right\rangle$ has de Sitter symmetries by construction.
- The regulator breaks some of them, but still gives a FLRW solution.
- The terms which breaks de Sitter group are UV dominated quantities which have practically no flow in the IR.
- Projecting on a de Sitter metric along the entire flow gives a good approximation.

The flow of the metric is reduced to the **flow of its Hubble constant**.

Table of Contents

Local potential approximation (LPA)

We compute the flow equation in the LPA by taking the ansatz

$$\Gamma_{\tilde{\kappa}}[\varphi,h] = -\int \mathrm{d}^D x \sqrt{-\tilde{g}} \left(\frac{Z(h)}{2} \tilde{g}_{\mu\nu} \partial^\mu \varphi_a \partial^\nu \varphi_a + N \tilde{U}_{\tilde{\kappa}}(\varphi_a, h) \right)$$

with $\tilde{g}$ the dS metric with $h = 1$. The h factors are hidden in Z and $\tilde{U}$.

It amounts to **discard higher derivative interactions**, which are expected to be subdominant in the infrared regime ($\kappa \ll h$). For constant φ, we compute the flow of U, the effective potential.

Flow equation

In the p-representation,

$$R_{\tilde{\kappa}}^{ab}(p,p';h) = \delta^{ab}\frac{\delta(p-p')}{p^2}Z(h)\left(\tilde{\kappa}^2 - p^2\right)\theta(\tilde{\kappa}^2 - p^2).$$

Then

$$N\dot{\tilde{U}}_{\tilde{\kappa}} = \beta(m_{l,\tilde{\kappa}}^2, \tilde{\kappa}) + (N-1)\beta(m_{t,\tilde{\kappa}}^2, \tilde{\kappa}).$$

Under the **small curvature** of the potential, in the **infrared** regime,

$$h^D\beta(m^2,\kappa) = \frac{h^D}{\Omega_{D+1}}\frac{\kappa^2}{\kappa^2+m^2}$$

M. Guilleux, J. Serreau ’15

Zero dimensional theory

The solution is a zero dimensional theory

$$e^{h^{-D}\Omega_{D+1}\mathscr{W}_\kappa(j,h)} = \int \mathrm{d}^N\hat{\varphi}\, e^{-h^{-D}\Omega_{D+1}\left(V_{in}(\hat{\varphi},h)+\frac{\kappa^2}{2}\hat{\varphi}^2-j\cdot\hat{\varphi}\right)}$$

with the initial conditions V_{in} that match the microscopic potential,

- It coincides with the equilibrium probability distribution in the stochastic formalism

  A. A. Starobinsky, J. Yokoyama '94
- It is the effective theory for the scalar field averaged over a Hubble patch at constant values of the field

Flow of the physical quantities

Taking as initial conditions

$$V_{in}(\hat{\varphi}, h) = N\left(\alpha - \frac{\beta}{2}h^2\right) + \frac{m^2 + \xi h^2}{2}\hat{\varphi}_a^2 + \frac{\lambda}{8N}(\hat{\varphi}_a^2)^2.$$

The minimization of the effective action gives

$$\begin{cases} \varphi_\kappa = \langle \hat{\varphi} \rangle \\ h_\kappa^2 = \dfrac{4N\alpha + 2\left(m^2 + \kappa^2\right)\langle \hat{\varphi}^2 \rangle + \frac{\lambda}{2N}\langle \hat{\varphi}^4 \rangle - 2\kappa^2\varphi^2}{N\beta - \xi \langle \hat{\varphi}^2 \rangle} \end{cases}$$

The expectation values are to be computed in the zero dimensional theory.

Approximations

A summary of our approximations so far :

- Semiclassical regime : $\dfrac{h_\kappa^2}{\beta} \ll 1$
- Infrared regime ($\rightarrow$ LPA) : $\kappa \ll h_\kappa$
- Small curvature : $\dfrac{m_{t/l,\kappa}^2}{h_\kappa^2} \ll 1$

Table of Contents

Gaussian theory

We look at cases where we can do analytical computations.
For a Gaussian ($\lambda = 0$) theory, $\varphi_\kappa = 0$ and

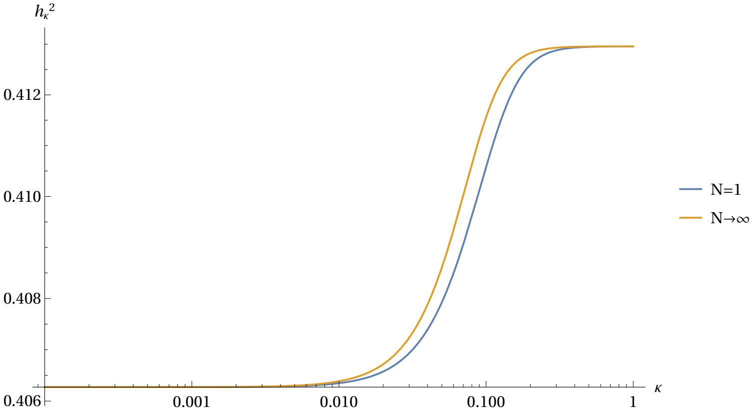
$$4\alpha - \beta h_\kappa^2 + \frac{2h_\kappa^4}{\Omega} - \frac{\xi h_\kappa^6}{\Omega \mu(h_\kappa)^2} = 0$$

with $\mu(h)^2 = m^2 + \xi h^2 + \kappa^2$.

- For minimally coupled fields ($\xi = 0$), h_κ has no flow.
- Depending on the sign of ξ, the Hubble constant is renormalized either positively or negatively.

Massless case

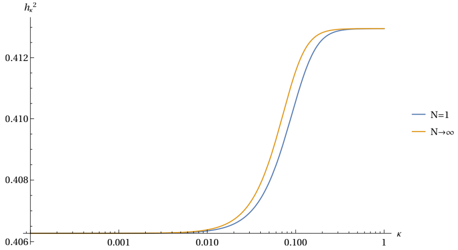
$$m = \xi = 0$$



Massless case

$$h_\infty^2 = \frac{\beta\Omega}{4}\left(1 - \sqrt{1 - \frac{32\alpha}{\beta^2\Omega}}\right)$$

$$h_0^2 = \frac{\beta\Omega}{4}\left(1 - \sqrt{1 - \frac{16\alpha}{\beta^2\Omega}}\right)$$



- The superhorizon modes of the massless scalar fields are greatly enhanced, drawing energy from the gravitational field
- The dynamical generation of a mass screens this effect, leading to a finite renormalization of the Hubble constant
- the asymptotic values can be computed exactly and only depend on α and β

Large N case

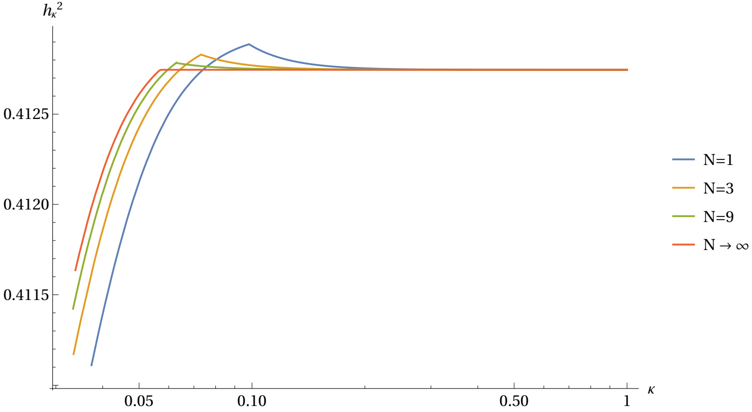
The massless and Gaussian cases forbid **symmetry breaking**. We can study it in the large N regime, having a (would-be) broken phase in the beginning of the flow.
The symmetry is always restored at a finite value of κ.

$$4\alpha - \beta h_\kappa^2 + 2\frac{\bar{z} - \mu^2}{\lambda}\left(\bar{z} + m^2 + \kappa^2\right) - 4\kappa^2\rho_\kappa = 0, \quad \rho = \frac{\varphi_a^2}{2N}$$

$$\bar{z} = m_{t,\kappa}^2 + \kappa^2 = \frac{\mu^2 + \lambda\rho}{2} + \sqrt{\left(\frac{\mu^2 + \lambda\rho}{2}\right)^2 + \frac{\lambda h^4}{2\Omega_{D+1}}}$$

Large N : (would-be) broken phase

The Goldstone bosons do not renormalize h_κ



Large N : symmetric phase

Interplay between λ and $\xi < 0$

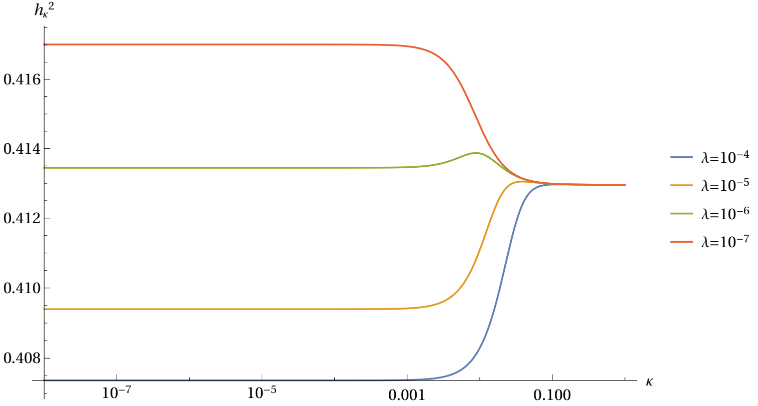


Table of Contents

Conclusion

The backreaction we studied is influenced by several phenomena :

- The mass generation screens the renormalization of the Hubble parameter
- Non minimal coupling between the scalar fields and gravitational field has a non trivial effect on the flow
- Goldstone modes do not contribute

Perspectives :

- Going beyond the local potential approximation
- Work in a more general FLRW spacetime