Black holes and Horndeski's theory

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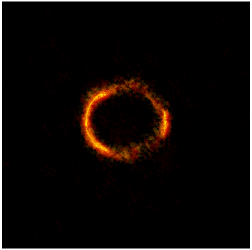
GPhys workshop
8/06/18

Outline

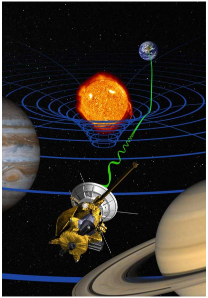
Context

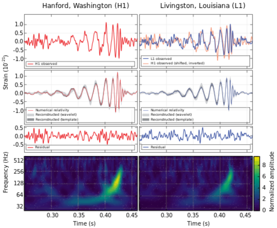
- General Relativity: best, "simplest" classical theory of gravity so far



Light deflection

Shapiro delay





Gravitational waves

...

Context

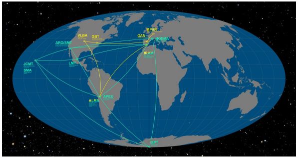
- But expected to break down above some energy scale

$\longrightarrow$ Tested against data from strong gravitational fields:

LIGO/VIRGO/LISA

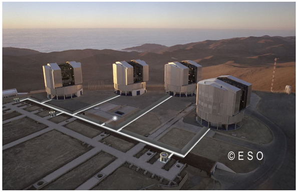
GW from coalescing compact objects

EVENT HORIZON TELESCOPE



*Pictures of the surrondings of Sgr A**

GRAVITY



*High precision astrometry of the bodies orbitting Sgr A**

Horndeski's theory

- Modified gravity: modify GR to account for observed deviations

Horndeski's theory

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Lagrangian formulation of GR

- $S_{GR}\left[g, m^a\right] = \int \left[R^{(g)} - 2\Lambda + \mathcal{L}_{matter}\left(m^a, \nabla m^a\right)\right] \sqrt{|\det g|} d^4x$
- Stationarity of $S_{GR} \longrightarrow G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}$

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Horndeski's theory

- $S_{\mathcal{H}}\left[g, \phi, m^a\right] = \int \left[\mathcal{L}^{(g,\phi)}_{nonminim} + \mathcal{L}_{matter}\left(m^a, \nabla m^a\right)\right] \sqrt{|\det g|}\, d^4x$
- Stationarity of $S_{\mathcal{H}}$

  $\longrightarrow$ most general 2^{nd} order Euler-Lagrange equations in g and ϕ

Cubic Galileon

Lagrangian formulation of GR

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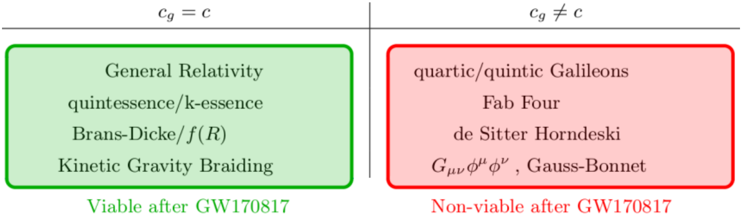
$$\mathcal{L}^{(g,\phi)}_{nonminim} = \zeta\left(R^{(g)} - 2\Lambda\right) + \left(-\eta + \gamma\Box\phi\right)\nabla_\mu\phi\nabla^\mu\phi$$

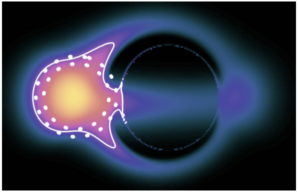
Cubic Galileon

- Emerges as a limit of an important brane model ("DGP" model)
- Consistent with GW170817:



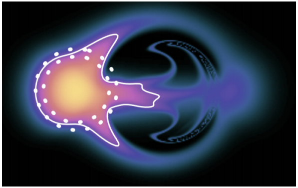
Ezquiaga & Zumalacárregui, Phys. Rev. Lett. 119, 251304 (2017)

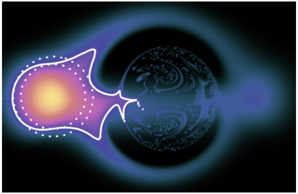
Predictions of GR



Kerr black hole

Boson star[1]





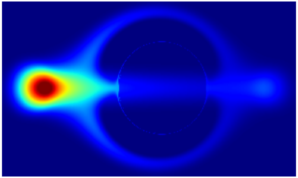
Black hole with scalar hair[2]

[1] Vincent, Meliani, Grandclément, Gourgoulhon & Straub, Class. Quantum Grav. 33, 105015 (2016)

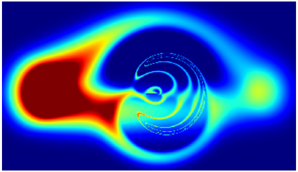
[2] Vincent, Gourgoulhon, Herdeiro & Radu, Phys. Rev. D 94, 084045 (2016)

using the libraries LORENE *and* KADATH *and the ray-tracing code* GYOTO

Predictions of GR



Regular black hole[3]



Rotating naked wormhole[3]

- *Sgr A** most likely a black hole

$\longrightarrow$ Focus on black holes in Cubic Galileon theory

[3] Lamy, Gourgoulhon, Paumard & Vincent, Class. Quantum Grav. 35, 115009 (2018)

No scalar hair theorems

- Different theory $\not\Rightarrow$ different black holes:

e.g. for Cubic Galileon, static spherically symmetric BH with $\phi(r)$
$\Longrightarrow$ Schwarzschild

[4] Babichev, Charmousis, Lehébel & Moskalets, JCAP09(2016)011

No scalar hair theorems

- Different theory $\not\Rightarrow$ different black holes:

e.g. for Cubic Galileon, static spherically symmetric BH with $\phi(r)$
$\Longrightarrow$ Schwarzschild

$\longrightarrow$ Introduce a linear time dependence[4]: $\phi = qt + \Psi(r)$

$\rightarrow$ Preserves spacetime symmetries

$\rightarrow$ Consistent with cosmological dynamics

$\rightarrow$ Yields BH different from GR ones

[4] Babichev, Charmousis, Lehébel & Moskalets, JCAP09(2016)011

Additional assumption

• Circularity: stationary axisymmetric spacetime with additional property of orthogonality

$$\iff \quad g_{\mu\nu}(r,\theta) = \begin{pmatrix} -N^2 + B^2\omega^2 r^2 \sin^2\theta & 0 & 0 & -\omega B^2 r^2 \sin^2\theta \\ 0 & A^2 & 0 & 0 \\ 0 & 0 & A^2 r^2 & 0 \\ -\omega B^2 r^2 \sin^2\theta & 0 & 0 & B^2 r^2 \sin^2\theta \end{pmatrix}$$

$\longrightarrow$ 4 unknown functions (instead of 10) in 2D

Problem

- Recall vacuum action of Cubic Galileon:

$$S_{\mathcal{H}}\left[g,\phi\right]=\int\left[\zeta\left(R^{(g)}-2\Lambda\right)+\left(-\eta+\gamma\Box\phi\right)\nabla_{\mu}\phi\nabla^{\mu}\phi\right]\sqrt{|\det g|}d^{4}x$$

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$\longrightarrow$ Derive the metric equations from $\frac{\delta S_{\mathcal{H}}}{\delta g_{\mu\nu}}$ + scalar equation from $\frac{\delta S_{\mathcal{H}}}{\delta\phi}$

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- Inject circular metric and scalar ansatz

$\longrightarrow$ Solve 5 coupled nonlinear PDE's in N, A, B, ω, Ψ

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- Set boundary conditions defining: - event horizon of a rotating BH
  - flat asymptotics

Spectral methods

- Discretization: consider the truncated decompositions onto standard basis functions

$$e.g.\quad A(r,\theta) = \sum_{i=0}^{N_r} \sum_{j=0}^{N_\theta} \tilde{A}_{ij} T_i(r) \cos(2j\theta)$$

[5] Grandclément, J. Comput. Phys. 229, 3334 (2010), http://kadath.obspm.fr/

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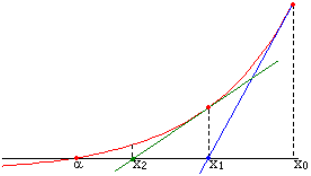
$\longrightarrow$ Transforms any system of PDE's into a nonlinear *algebraic* system

$\longrightarrow$ Use Newton-Raphson algorithm implemented in KADATH library[5]



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Initial configuration

- Required to be relatively close to the exact solution

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Initial configuration

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- Build static spherically symmetric solution

$\longrightarrow$ Non trivial system of ODE's[6]

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$\longrightarrow$ Non trivial system of ODE's[6]

- Use static BH as initial configuration to obtain a very slowly rotating BH

- Use slowly rotating solution as initial configuration to obtain a less slowly rotating BH...

[6] Babichev, Charmousis, Lehébel & Moskalets, JCAP09(2016)011

Summary

- Observations of *Sgr A** provide new tests of GR

- Horndeski theory such as Cubic Galileon may account for deviations from GR

- A numerical BH would reveal what deviations could be accounted for